Polynomial Exercise 3

Question 1

Divide the polynomial $p(x)$ by the polynomial $g(x)$ and find the quotient and remainder in each of the following:

(i) $p(x) = x^3 - 3x^2 + 5x - 3$, $g(x) = x^2 - 2$
(ii) $p(x) = x^4 - 3x^2 + 4x + 5$, $g(x) = x^2 + 1 - x$
(iii) $p(x) = x^4 - 5x + 6$, $g(x) = 2 - x^2$

Answer

(i) $p(x) = x^3 - 3x^2 + 5x - 3$, $g(x) = x^2 - 2$

	$x-3$
$x^2 - 2$	$x^3 - 3x^2 + 5x - 3$
	$x^3 \quad -2x$
	$- \quad +$
	$-3x^2 + 7x - 3$
	$-3x^2 \quad +6$
	$+ \quad -$
	$7x - 9$

Quotient = $x-3$ and remainder $7x - 9$

(ii) $p(x) = x^4 - 3x^2 + 4x + 5$, $g(x) = x^2 + 1 - x$

	$x^2 + x - 3$
$x^2 + 1 - x$	$x^4 \quad - 3x^2 + 4x + 5$
	$x^4 - x^3 + x^2$
	$- \quad + \quad -$
	$x^3 - 4x^2 + 4x + 5$
	$x^3 - x^2 + x$
	$- \quad + \quad -$
	$-3x^2 + 3x + 5$
	$-3x^2 + 3x - 3$
	$+ \quad - \quad +$
	8

Quotient = $x^2 + x - 3$ and remainder 8

(iii) $p(x) = x^4 – 5x + 6$, $g(x) = 2 – x^2$

	$-x^2 - 2$
$-x^2 + 2$	$x^4 \quad - 5x + 6$
	$x^4 - 2x^2$
	$- \quad +$
	$2x^2 - 5x + 6$
	$2x^2 \quad - 4$
	$- \quad +$
	$-5x + 10$

Quotient = $-x^2 - 2$ and remainder $-5x + 10$

Question 2.

Check whether the first polynomial is a factor of the second polynomial by dividing the second polynomial by the first polynomial:

(i) $t^2 – 3$, $2t^4 + 3t^3 – 2t^2 – 9t – 12$

(ii) $x^2 + 3x + 1$, $3x^4 + 5x^3 – 7x^2 + 2x + 2$

(iii) $x^3 – 3x + 1$, $x^5 – 4x^3 + x^2 + 3x + 1$

Answer

(i) $t^2 – 3$, $2t^4 + 3t^3 – 2t^2 – 9t – 12$

	$2t^2 + 3t + 4$
$t^2 - 3$	$2t^4 + 3t^3 - 2t^2 - 9t - 12$
	$2t^4 \quad -6t^2$ $- \quad +$
	$3t^3 + 4t^2 - 9t - 12$
	$3t^3 \quad -9t$ $- \quad +$
	$4t^2 \quad -12$
	$4t^2 \quad -12$ $- \quad +$
	0

So
$t^2 - 3$ exactly divides $2t^4 + 3t^3 - 2t^2 - 9t - 12$ leaving no remainder. Hence, it is a factor of $2t^4 + 3t^3 - 2t^2 - 9t - 12$.

(ii) $x^2 + 3x + 1$, $3x^4 + 5x^3 - 7x^2 + 2x + 2$

	$3x^2 + 4x + 2$
$x^2 + 3x + 1$	$3x^4 + 5x^3 - 7x^2 + 2x + 2$
	$3x^4 + 9x^3 - 3x^2$ $- \quad - \quad +$
	$-4x^3 - 10x^2 + 2x + 2$
	$-4x^3 - 12x^2 - 4x$ $- \quad + \quad +$
	$2x^2 + 6x + 2$
	$2x^2 + 6x + 2$ $- \quad - \quad -$
	0

$x^2 + 3x + 1$ exactly divides $3x^4 + 5x^3 - 7x^2 + 2x + 2$ leaving no remainder. Hence, it is factor of $3x^4 + 5x^3 - 7x^2 + 2x + 2$.
(iii) $x^3 - 3x + 1$, $x^5 - 4x^3 + x^2 + 3x + 1$

$x^3 - 3x + 1$ didn't divides exactly $x^5 - 4x^3 + x^2 + 3x + 1$ and leaves 2 as remainder. Hence, it not a factor of $x^5 - 4x^3 + x^2 + 3x + 1$.

Question 3.
Obtain all other zeroes of $3x^4 + 6x^3 - 2x^2 - 10x - 5$, if two of its zeroes are $\sqrt{(5/3)}$ and $-\sqrt{(5/3)}$.

Answer

$p(x) = 3x^4 + 6x^3 - 2x^2 - 10x - 5$
Since the two zeroes are $\sqrt{(5/3)}$ and $-\sqrt{(5/3)}$.

Therefore

$[x-\sqrt{(5/3)}]$ and $[x+\sqrt{(5/3)}]$ are factors of the polynomial p(x)

So $[x-\sqrt{(5/3)}]\ [x+\sqrt{(5/3)}] = (x^2 - 5/3)$ is a factor of the polynomial p(x)

Let's divide the p(x) by $(x^2 - 5/3)$ to get remaining factors

	$3x^2 + 6x + 3$
$x^2 - 5/3$	$3x^4 + 6x^3 - 2x^2 - 10x - 5$
	$3x^4 \quad -5x^2$
	$- \quad +$
	$6x^3 \quad +3x^2 \quad -10x \quad -5$
	$6x^3 \quad -10x$
	$- \quad +$
	$3x^2 \quad -5$
	$3x^2 \quad -5$
	$- \quad +-$
	0

So
$P(x) = 3x^4 + 6x^3 - 2x^2 - 10x - 5$
$= (3x^2 + 6x + 3)(x^2 - 5/3)$
$= 3(x^2 + 2x + 1)(x^2 - 5/3)$

We factorize $x^2 + 2x + 1$
$= (x + 1)^2$
Therefore, its zero is given by $x + 1 = 0$
$x = -1$
So the other 2 zeroes are $x = -1$.
Hence, the zeroes of the given polynomial are $\sqrt{(5/3)}$ and $-\sqrt{(5/3)}$, - 1 and - 1.

Question 4.

On dividing $x^3 - 3x^2 + x + 2$ by a polynomial $g(x)$, the quotient and remainder were $x - 2$ and $-2x + 4$, respectively. Find $g(x)$.

Answer

Here in the given question,

Dividend = $x^3 - 3x^2 + x + 2$
Quotient = $x - 2$
Remainder = $-2x + 4$
Divisor = $g(x)$
We know that,
Dividend = Quotient × Divisor + Remainder
$\Rightarrow x^3 - 3x^2 + x + 2 = (x - 2) \times g(x) + (-2x + 4)$

$x^3 - 3x^2 + x + 2 - (-2x + 4) = (x - 2) \times g(x)$
$x^3 - 3x^2 + 3x - 2 = (x - 2) \times g(x)$
$g(x) = (x^3 - 3x^2 + 3x - 2)/ (x - 2)$

	$x^2 - x + 1$
$x - 2$	$x^3 - 3x^2 + 3x - 2$
	$x^3 \;\; -2x^2$ $- \;\; +$
	$-x^2 \;\; +3x \;\; -2$
	$-x^2 \;\; +2x$ $+ \;\; -$
	$x-2$
	$x - 2$ $- \;\; +$
	0

∴ $g(x) = (x^2 - x + 1)$

Question 5

Give examples of polynomial $p(x)$, $g(x)$, $q(x)$ and $r(x)$, which satisfy the division algorithm and
(i) degree $p(x)$ = degree $q(x)$
(ii) degree $q(x)$ = degree $r(x)$
(iii) degree $r(x) = 0$

Answer

According to the division algorithm, if p(x) and g(x) are two polynomials

with g(x) ≠ 0, then we can find polynomials q(x) and r(x) such that

p(x) = g(x) × q(x) + r(x), --(A)

where r(x) = 0 or degree of r(x) < degree of g(x)

.(i)

degree $p(x)$ = degree $q(x)$

From equation (A), then r(x)=0 and q(x) =constant term

Here Let us assume the division of $9x^2 + 6x + 3$ by 3
Here, $p(x) = 9x^2 + 6x + 3$
$g(x) = 3$
$q(x) = 3x^2 + 2x + 1$
$r(x) = 0$
Degree of $p(x)$ and $q(x)$ is same i.e. 2.
Checking for division algorithm,

$p(x) = g(x) \times q(x) + r(x)$
Or, $9x^2 + 6x + 3 = 3 \times (3x^2 + 2x + 1)$
Hence, division algorithm is satisfied.

(ii)

Now degree $q(x)$ = degree $r(x)$
Let us assume the division of $x^3 + x$ by x^2,
Here, $p(x) = x^3 + x$
$g(x) = x^2$
$q(x) = x$ and $r(x) = x$
Clearly, the degree of $q(x)$ and $r(x)$ is the same i.e., 1.
Checking for division algorithm,
$p(x) = g(x) \times q(x) + r(x)$
$x^3 + x = (x^2) \times x + x$
$x^3 + x = x^3 + x$
Thus, the division algorithm is satisfied.

(iii)

degree $r(x) = 0$

Let us assume the division of $x^3 + 5$ by x^2.
Here, $p(x) = x^3 + 5$
$g(x) = x^2$
$q(x) = x$ and $r(x) = 5$
Clearly, the degree of $r(x)$ is 0.
Checking for division algorithm,
$p(x) = g(x) \times q(x) + r(x)$
$x^3 + 5 = (x^2) \times x + 5$
$x^3 + 5 = x^3 + 5$
Thus, the division algorithm is satisfied.