Real Number Exercise 2,3 and 4

Important Tip
How to Factorize the Composite Numbers?

1) Few things we need to remember,
If the number is even, then it will be divisible by 2
If the sums of its digits is divisible by 3, then it is divisible by 3
if the end of the number is 0 or 5, then it is divisible by 5
2) We have start with the small prime number with the rules given in step 1. Once we find the quotient, repeat the same process for the quotient. The last quotient will be a prime number itself

Example

Suppose the composite Number is 168
1) Now 168 is even number, so we know it will get divided by 2
168 / 2 = 84
2) Again 84 is even number, so we know it will get divided by 2
84 / 2 = 42
3) Again 42 is even number, so we know it will get divided by 2
42 / 2 = 21
4) Now sum of digits of 21 is 3,so we know we can divide it by 3
21 / 3 = 7
5)7 is a prime Number

So prime factors = 2 X 2 X 2 X 3 X 7

Here are the prime factors of the composite numbers between 1 and 30.
4 = 2 X 2
6 = 3 X 2
8 = 2 X 2 X 2
9 = 3 X 3
10 = 5 X 2
12 = 3 X 2 X 2
14 = 7 X 2
15 = 5 X 3
16 = 2 X 2 X 2 X 2
18 = 3 X 3 X 2
20 = 5 X 2 X 2
21= 3 X 7
22=2 X 11
24= 2 X 2 X2 X3
25= 5 X 5

26= 2 X 13
27= 3 X 3 X 3
28= 2 X 2 X7
30= 2 X 3 X 5

Question 1

Express each number as product of its prime factors:

(i) 140

(ii) 156

(iii) 3825

(iv) 5005

(v) 7429

Answer

(i) 140 = 2 × 2 × 5 × 7 = 2^2 × 5 × 7

(ii) 156 = 2 × 2 × 3 × 13 = 2^2 × 3 × 13

(iii) 3825 = 3 × 3 × 5 × 5 × 17 = 3^2 × 5^2 × 17

(iv) 5005 = 5 × 7 × 11 × 13

(v) 7429 = 17 × 19 × 23

Question 2

Find the LCM and HCF of the following pairs of integers and verify that LCM × HCF = product of the two numbers.

(i) 26 and 91

(ii) 510 and 92

(iii) 336 and 54

Answer

(i)

Prime Factorization of the Numbers

26 = 2 × 13

91 =7 × 13

HCF = 13

LCM =2 × 7 × 13 =182

Product of two numbers 26 × 91 = 2366
Product of HCF and LCM 13 × 182 = 2366
Hence, product of two numbers = product of HCF × LCM

(ii)

Prime Factorization of the Numbers

510 = 2 × 3 × 5 × 17
92 =2 × 2 × 23
HCF = 2
LCM =2 × 2 × 3 × 5 × 17 × 23 = 23460
Product of two numbers 510 × 92 = 46920
Product of HCF and LCM 2 × 23460 = 46920
Hence, product of two numbers = product of HCF × LCM

(iii) Prime Factorization of the Numbers

336 = 2 × 2 × 2 × 2 × 3 × 7
54 = 2 × 3 × 3 × 3
HCF = 2 × 3 = 6
LCM = 2 × 2 × 2 × 2 × 3 × 3 × 3 × 7 =3024
Product of two numbers 336 × 54 =18144
Product of HCF and LCM 6 × 3024 = 18144
Hence, product of two numbers = product of HCF × LCM.

Question 3

Find the LCM and HCF of the following integers by applying the prime factorization method.

(i) 12, 15 and 21

(ii) 17, 23 and 29

(iii) 8, 9 and 25

Answer

(i) Prime Factorization of the Numbers

12 = 2 × 2 × 3
15 =3 × 5
21 =3 × 7
HCF = 3
LCM = 2 × 2 × 3 × 5 × 7 = 420

(ii) Prime Factorization of the Numbers

17 = 1 × 17
23 = 1 × 23
29 = 1 × 29
HCF = 1
LCM = 1 × 17 × 19 × 23 = 11339

(iii) Prime Factorization of the Numbers

8 =1 × 2 × 2 × 2
9 =1 × 3 × 3
25 =1 × 5 × 5
HCF =1
LCM = 1 × 2 × 2 × 2 × 3 × 3 × 5 × 5 = 1800

Question 4

Given that HCF (306, 657) = 9, find LCM (306, 657).

Answer

We have the formula that
Product of LCM and HCF = product of number
LCM × 9 = 306 × 657
Divide both side by 9 we get
LCM = (306 × 657) / 9 = 22338

Question 5

Check whether 6^n can end with the digit 0 for any natural number n.

Answer

If any digit has last digit 10 that means it is divisible by 10 and the factors of 10 = 2 × 5.

So value 6^n should be divisible by 2 and 5

Now $(2\times3)^n$ is divisible by 2 and 3 for sure but not divisible by 5. So it can not end with 0.

Question 6

Explain why 7 × 11 × 13 + 13 and 7 × 6 × 5 × 4 × 3 × 2 × 1 + 5 are composite numbers.

Answer

7 × 11 × 13 + 13
Taking 13 common, we get
13 (7 x 11 +1)
13(77 + 1)
13 (78)
It is product of two numbers and both numbers are more than 1 so it is a composite number.

7 × 6 × 5 × 4 × 3 × 2 × 1 + 5
Taking 5 common, we get
5(7 × 6 × 4 × 3 × 2 × 1 +1)
5(1008 + 1)
5(1009)
It is product of two numbers and both numbers are more than 1 so it is a composite number.

Question 7

There is a circular path around a sports field. Sonia takes 18 minutes to drive one round of the field, while Ravi takes 12 minutes for the same. Suppose they both start at the same point and at the same time, and go in the same direction. After how many minutes will they meet again at the starting point?

Answer

It is an exercise for LCM.They will be meet again after LCM of both values at the starting point.
$18 = 2 \times 3 \times 3$
$12 = 2 \times 2 \times 3$
LCM $= 2 \times 2 \times 3 \times 3 = 36$
Therefore, they will meet together at the starting point after 36 minutes.

Exercise 1.3

Question 1

Prove that √5 is irrational.

Answer

Let take √5 as rational number
If a and b are two co prime number and b is not equal to 0.
We can write $\sqrt{5} = a/b$
Multiply by b both side we get
$b\sqrt{5} = a$
To remove root, squaring on both sides, we get
$5b^2 = a^2$ … **(i)**

Therefore, 5 divides a^2 and according to theorem of rational number, for any prime number p which is divides a^2 then it will divide a also.
That means 5 will divide a. So we can write
$a = 5k$
Putting value of a in equation **(i)** we get
$5b^2 = (5k)^2$
$5b^2 = 25k^2$
Divide by 5 we get
$b^2 = 5k^2$

Similarly, we get that b will divide by 5
and we have already get that a is divide by 5

but *a* and *b* are co prime number. so it contradicts.
Hence √5 is not a rational number, it is irrational.

Question 2

Prove that 3 + 2√5 is irrational.

Answer

Let take that 3 + 2√5 is a rational number.
So we can write this number as
$3 + 2\sqrt{5} = a/b$
Here a and b are two co prime number and b is not equal to 0
Subtract 3 both sides we get
$2\sqrt{5} = a/b - 3$
$2\sqrt{5} = (a-3b)/b$
Now divide by 2, we get
$\sqrt{5} = (a-3b)/2b$
Here *a* and *b* are integer so $(a-3b)/2b$ is a rational number so √5 should be a rational number but √5 is a irrational number so it contradicts.
Hence, 3 + 2√5 is a irrational number.

Question 3

Prove that the following are irrationals:
(i) $1/\sqrt{2}$ (ii) $7\sqrt{5}$ (iii) $6 + \sqrt{2}$

Answer

(i) Let take that $1/\sqrt{2}$ is a rational number.
So we can write this number as
$1/\sqrt{2} = a/b$
Here *a* and *b* are two co prime number and *b* is not equal to 0
Multiply by √2 both sides we get
$1 = (a\sqrt{2})/b$
Now multiply by *b*
$b = a\sqrt{2}$
divide by a we get
$b/a = \sqrt{2}$

Here *a* and *b* are integer so b/a is a rational number so √2 should be a rational number But √2 is a irrational number so it contradicts.
Hence, 1/√2 is a irrational number

(ii) Let take that 7√5 is a rational number.
So we can write this number as
7√5 = *a*/*b*
Here *a* and *b* are two co prime number and *b* is not equal to 0
Divide by 7 we get
√5 = *a*/(7*b*)
Here *a* and *b* are integer so *a*/7*b* is a rational number so √5 should be a rational number but √5 is a irrational number so it contradicts.
Hence, 7√5 is a irrational number.

(iii) Let take that 6 + √2 is a rational number.
So we can write this number as
6 + √2 = *a*/*b*
Here a and b are two co prime number and b is not equal to 0
Subtract 6 both side we get
√2 = *a*/*b* – 6
√2 = (*a*-6*b*)/*b*
Here *a* and *b* are integer so (a-6*b*)/*b* is a rational number so √2 should be a rational number.

But √2 is a irrational number so it contradicts.
Hence, 6 + √2 is a irrational number.

Exercise 1.4

Question 1

Without actually performing the long division, state whether the following rational numbers will have a terminating decimal expansion or a non-terminating repeating decimal
expansion:
(i) 13/3125

(ii) 17/8

(iii) 64/455

(iv) 15/1600

(v) 29/343

(vi) $23/2^3 \times 5^2$

(vii) $129/2^2 \times 5^7 \times 7^5$

(viii) 6/15

(ix) 35/50

(x) 77/210

Answer

We know that for terminating decimal expansion of a rational number of form *p*/q ,*q* must be of the form $2^m \times 5^n$.

S.no	Rational Number	Denominator Factorization	Terminating/Non terminating
i)	13/3125	Factorize the denominator we get $3125 = 5 \times 5 \times 5 \times 5 \times 5 = 5^5$	As denominator is in form of 5^m so it is terminating.
ii)	17/8	Factorize the denominator we get $8 = 2 \times 2 \times 2 = 2^3$	As denominator is in form of 2^m so it is terminating.
iii)	64/455	Factorize the denominator we get $455 = 5 \times 7 \times 13$	There are 7 and 13 also in denominator so denominator is not in form of $2^m \times 5^n$.

			so it is not terminating.
iv)	15/1600	Factorize the denominator we get $1600 = 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 5 \times 5$ $= 2^6 \times 5^2$	As denominator is in form of $2^m \times 5^n$ Hence it is terminating
v)	29/343	Factorize the denominator we get $343 = 7 \times 7 \times 7 = 7^3$	There are 7 also in denominator so denominator is not in form of $2^m \times 5^n$ Hence it is non-terminating
vi)	$23/(2^3 \times 5^2)$		Denominator is in form of $2^m \times 5^n$ Hence it is terminating.
vii)	129/ $(2^2 \times 5^7 \times 7^5)$		Denominator has 7 in denominator so denominator is not in form of $2^m \times 5^n$ Hence it is none terminating.
viii)	6/15	divide nominator and denominator both by 3 we get 2/5	Denominator is in form of 5^m so it is terminating.
ix)	35/50	divide denominator and nominator both by 5 we get 7/10 Factorize the denominator we get $10 = 2 \times 5$	So denominator is in form of $2^m \times 5^n$ so it is terminating.

x)	77/210	simplify it by dividing nominator and denominator both by 7 we get 11/30 Factorize the denominator we get $30=2 \times 3 \times 5$	Denominator has 3 also in denominator so denominator is not in form of $2^m \times 5^n$ Hence it is none terminating.

Question 2

Write down the decimal expansions of those rational numbers in Question 1 above which have terminating decimal expansions.

Answer

(i) $13/3125 = 13/5^5 = 13\times2^5/5^5\times2^5 = 416/10^5 = 0.00416$

(ii) $17/8 = 17/2^3 = 17\times5^3/2^3\times5^3 = 17\times5^3/10^3 = 2125/10^3 = 2.125$

(iv) $15/1600 = 15/2^4\times10^2 = 15\times5^4/2^4\times5^4\times10^2 = 9375/10^6 = 0.009375$

(vi) $23/2^35^2 = 23\times5^3\times2^2/2^3\ 5^2\times5^3\times2^2 = 11500/10^5 = 0.115$

(viii) $6/15 = 2/5 = 2\times2/5\times2 = 4/10 = 0.4$

(ix) $35/50 = 7/10 = 0.7$.

Question 3

The following real numbers have decimal expansions as given below. In each case, decide whether they are rational or not. If they are rational, and of the form *p*, *q* you say about the prime factors of *q*?

(i) 43.123456789

(ii) 0.120120012000120000...

(iii) 43.123456789

Answer

(i) Since this number has a terminating decimal expansion, it is a rational number of the form *p*/q, and *q* is of the form $2^m \times 5^n$.

(ii) The decimal expansion is neither terminating nor recurring. Therefore, the given number is an irrational number.
(iii) Since the decimal expansion is non-terminating recurring, the given number is a rational number of the form *p*/q, and *q* is not of the form $2^m \times 5^n$.