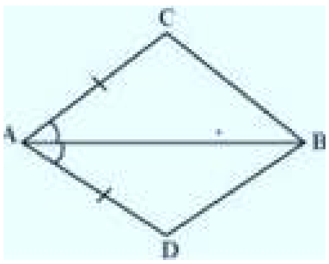
NCERT solutions of Triangles part 1

Question 1
In quadrilateral ACBD, AC = AD and AB bisects ∠A (see below figure). Show that ΔABC ≅ ΔABD. What can you say about BC and BD?



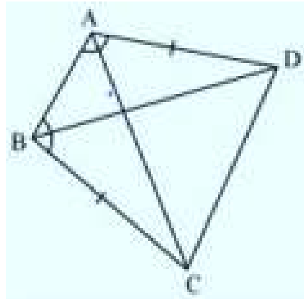
Answer

<u>Given</u>,
AC = AD and AB bisects ∠A
<u>To prove</u>,
ΔABC ≅ ΔABD
<u>Proof</u>,
In ΔABC and ΔABD,
AB = AB (Common)
AC = AD (Given)
∠CAB = ∠DAB (AB is bisector)
By SAS (Side-Angle-Side) congruence condition.

Therefore, ΔABC ≅ ΔABD.

Now from CPCT, we know that
BC=BD

Question 2
ABCD is a quadrilateral in which AD = BC and ∠DAB = ∠CBA (see Fig. 7.17). Prove that
(i) ΔABD ≅ ΔBAC
(ii) BD = AC
(iii) ∠ABD = ∠BAC.

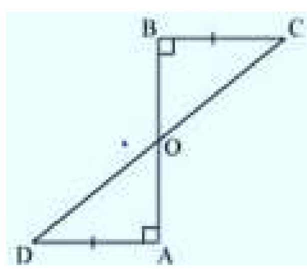


Answer

Given,
AD = BC and ∠DAB = ∠CBA

(i) In ΔABD and ΔBAC,
AB = BA (Common)
∠DAB = ∠CBA (Given)
AD = BC (Given)
By SAS congruence condition.
So, ΔABD ≅ ΔBAC
(ii) Since, ΔABD ≅ ΔBAC
Therefore BD = AC by CPCT
(iii) Since, ΔABD ≅ ΔBAC
Therefore ∠ABD = ∠BAC by CPCT

Question 3
AD and BC are equal perpendiculars to a line segment AB (see below figure). Show that CD bisects AB.



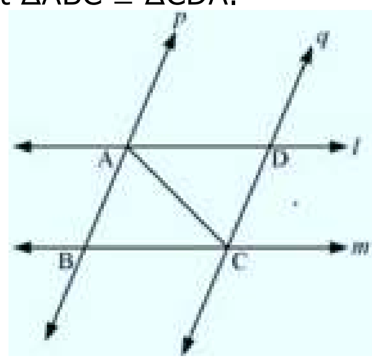
Answer

Given,
AD and BC are equal perpendiculars to AB.
To prove,
CD bisects AB
Proof,
In ΔAOD and ΔBOC,
∠A = ∠B (As Perpendicular)
∠AOD = ∠BOC (Vertically opposite angles)
AD = BC (Given)

By AAS(Angle-Angle-Side) congruence condition.

So, ΔAOD ≅ ΔBOC
Now by CPCT
AO = OB
So CD bisects AB.

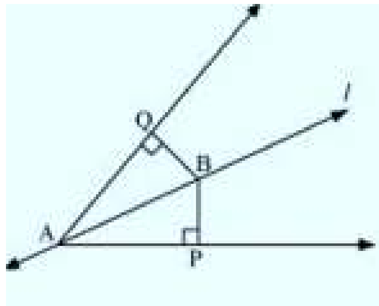
Question 4
l and m are two parallel lines intersected by another pair of parallel lines p and q (see below figure). Show that ΔABC ≅ ΔCDA.



Answer

<u>Given</u>,
l || m and p || q
<u>To prove</u>,
ΔABC ≅ ΔCDA
<u>Proof,</u>
In ΔABC and ΔCDA,
∠BCA = ∠DAC (Alternate interior angles)
AC = CA (Common)
∠BAC = ∠DCA (Alternate interior angles)
By ASA(Angle-Side-Angle) congruence condition.
So, ΔABC ≅ ΔCDA

Question 5
Line l is the bisector of an angle ∠A and B is any point on l. BP and BQ are perpendiculars from B to the arms of ∠A (see below figure). Show that:
(i) ΔAPB ≅ ΔAQB
(ii) BP = BQ or B is equidistant from the arms of ∠A.

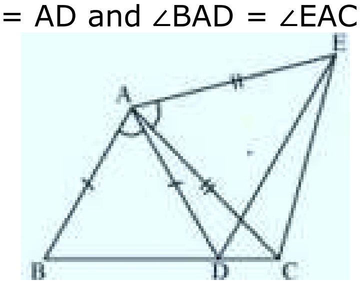


Answer

Given,
l is the bisector of an angle ∠A.
BP and BQ are perpendiculars.

(i) In ΔAPB and ΔAQB,
∠P = ∠Q (Right angles)
∠BAP = ∠BAQ (l is bisector)
AB = AB (Common)
Therefore, ΔAPB ≅ ΔAQB by AAS congruence condition.
(ii) BP = BQ by CPCT. Therefore, B is equidistant from the arms of ∠A.

Question 6
In below figure, AC = AE, AB = AD and ∠BAD = ∠EAC. Show that BC = DE.



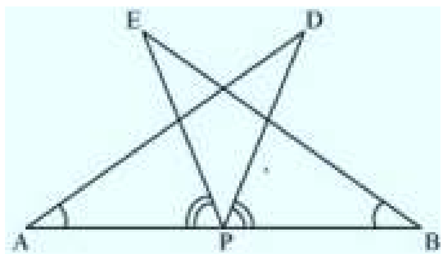
Answer

Given,
AC = AE, AB = AD and ∠BAD = ∠EAC
To Prove,
BC = DE
Proof,
∠BAD = ∠EAC
By Adding ∠DAC both sides
∠BAD + ∠DAC = ∠EAC + ∠DAC
∠BAC = ∠EAD
In ΔABC and ΔADE,
AC = AE (Given)

∠BAC = ∠EAD
AB = AD (Given)
By SAS (Side -Angle-Side) congruence condition.

So, ΔABC ≅ ΔADE
By CPCT
BC = DE

Question 7
AB is a line segment and P is its mid-point. D and E are points on the same side of AB such that ∠BAD = ∠ABE and ∠EPA = ∠DPB (see below figure). Show that
(i) ΔDAP ≅ ΔEBP
(ii) AD = BE



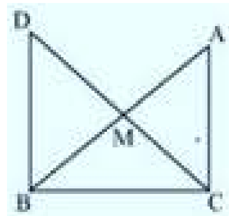
Answer

Given,
P is mid-point of AB.
∠BAD = ∠ABE and ∠EPA = ∠DPB

(i) ∠EPA = ∠DPB
By Adding ∠DPE both sides
∠EPA + ∠DPE = ∠DPB + ∠DPE
∠DPA = ∠EPB
In ΔDAP ≅ ΔEBP,
∠DPA = ∠EPB
AP = BP (P is mid-point of AB)
∠BAD = ∠ABE (Given)
By ASA congruence condition.
So, ΔDAP ≅ ΔEBP
(ii) By CPCT
AD = BE

Question 8
In right triangle ABC, right angled at C, M is the mid-point of hypotenuse AB. C is joined to M and produced to a point D such that DM = CM. Point D is joined to point B (see below figure). Show that:
(i) ΔAMC ≅ ΔBMD
(ii) ∠DBC is a right angle.

(iii) ΔDBC ≅ ΔACB
(iv) CM = 1/2 AB



Answer

Given,
∠C = 90°, M is the mid-point of AB and DM = CM

(i) In ΔAMC and ΔBMD,
AM = BM (M is the mid-point)
∠CMA = ∠DMB (Vertically opposite angles)
CM = DM (Given)
By SAS(Side-Angle-Side) congruence condition
So, ΔAMC ≅ ΔBMD.

(ii)By CPCT
∠ACM = ∠BDM
Therefore, AC || BD as alternate interior angles are equal.
Now,
∠ACB + ∠DBC = 180° (co-interiors angles)
90° + ∠B = 180°
∠DBC = 90°

(iii) In ΔDBC and ΔACB,
BC = CB (Common)
∠ACB = ∠DBC (Right angles)
DB = AC (by CPCT, already proved)
By (Side-Angle-Side) congruence condition.
So, ΔDBC ≅ ΔACB

(iv) DC = AB (ΔDBC ≅ ΔACB)
DM = CM = AM = BM (M is mid-point)
DM + CM = AM + BM
CM + CM = AB
CM = 1/2AB