Heron Formula Exercise 2

Question 1

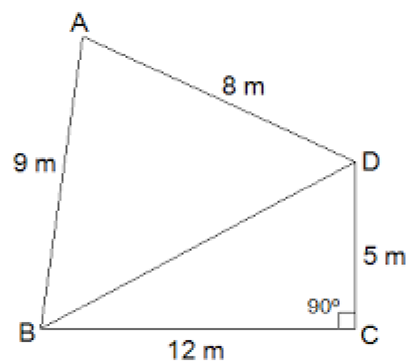
A park, in the shape of a quadrilateral ABCD, has ∠C = 90º, AB = 9 m, BC = 12 m, CD = 5 m and AD = 8 m. How much area does it occupy?

Answer

Given in the question

∠C = 90º, AB = 9 m, BC = 12 m, CD = 5 m and AD = 8 m

BD is joined.



Area of the quadrilateral ABCD can be found using the area of the separate triangle and then adding up

In ΔBCD,

By applying Pythagoras theorem,

$BD^2 = BC^2 + CD^2$

$BD^2 = 12^2 + 5^2$

$BD^2 = 169$

BD = 13 m

Area of ΔBCD=Area of right angle triangle= (1/2) × base × Height

So Area of ΔBCD = 1/2 × 12 × 5 = 30 m^2

Now,

Semi perimeter of ΔABD(s) = (8 + 9 + 13)/2 m = 30/2 m = 15 m

Using heron's formula,

Area of ΔABD = √s (s-a) (s-b) (s-c)

= √15(15 - 13) (15 - 9) (15 - 8) m^2

$= \sqrt{15 \times 2 \times 6 \times 7}\ m^2$

$= 6\sqrt{35}\ m^2 = 35.5\ m^2$ (approx)

Area of quadrilateral ABCD = Area of ΔBCD + Area of ΔABD = 30 m^2 + 35.5m^2 = 65.5m^2

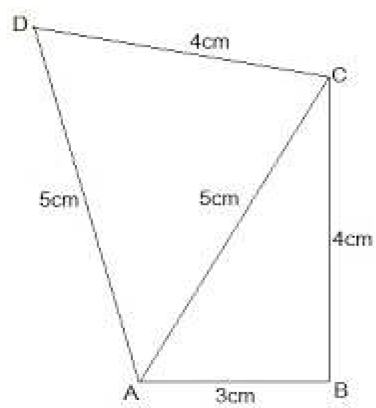
Question 2

Find the area of a quadrilateral ABCD in which AB = 3 cm, BC = 4 cm, CD = 4 cm, DA = 5 cm and AC = 5 cm.

Answer

Given in the question

AB = 3 cm, BC = 4 cm, CD = 4 cm, DA = 5 cm and AC = 5 cm



Area of the quadrilateral ABCD can be found using the area of the separate triangle and then adding up

In ΔABC,
By applying Pythagoras theorem,
$AC^2 = AB^2 + BC^2$
$5^2 = 3^2 + 4^2$
25 = 25
Thus, ΔABC is a right angled at B.

Area of ΔABC=Area of right angle triangle= (1/2) × base × Height

Area of ΔABC = 1/2 × 3 × 4 = 6 cm^2

Now,

Semi perimeter of ΔACD(s) = (5 + 5 + 4)/2 cm = 14/2 cm = 7 m

Using heron's formula,

Area of ΔABD = √s (s-a) (s-b) (s-c)

= √7(7 - 5) (7 - 5) (7 - 4) cm^2

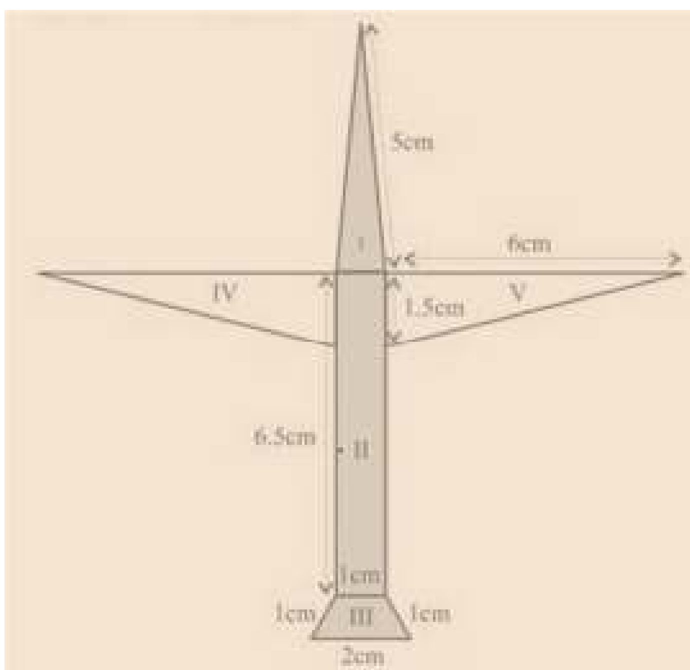
= √7 × 2 × 2 × 3 cm^2

= 2√21 cm^2 = 9.17 cm^2

Now Area of quadrilateral ABCD = Area of ΔABC + Area of ΔABD = 6 cm^2 + 9.17 cm^2 = 15.17 cm^2

Question 3

Radha made a picture of an aeroplane with coloured paper as shown in below figure. Find the total area of the paper used.



Answer

Area of the aeroplane can be find using the area of the individual triangles and then summing them to get the total area

Area of triangle I

Length of the sides of the triangle section I = 5cm, 1cm and 5cm
Perimeter of the triangle = 5 + 5 + 1 = 11cm
Semi perimeter = 11/2 cm = 5.5cm
Using heron's formula,
Area of section I $= \sqrt{s (s-a) (s-b) (s-c)}$
$= \sqrt{5.5(5.5 - 5) (5.5 - 5) (5.5 - 1)}\ cm^2$
$= \sqrt{5.5 \times 0.5 \times 0.5 \times 4.5}\ cm^2$
$= 0.75\sqrt{11}\ cm^2 = 0.75 \times 3.317 cm^2 = 2.488 cm^2$

Area of the rectangle II

Length of the sides of the rectangle of section II = 6.5cm and 1cm
Area of section II = $6.5 \times 1\ cm^2 = 6.5\ cm^2$

Area of the trapezium III

Section III is an isosceles trapezium which is divided into 3 equilateral of side 1cm each.
Area of the trapezium = $3 \times \sqrt{3}/4 \times 1^2\ cm^2 = 1.3\ cm^2$

Area of triangle IV

Section IV is right angled triangles with base 6cm and height 1.5cm
Area of region IV = $1/2 \times 6 \times 1.5 cm^2 = 4.5 cm^2$

Area o f triangle V

Section IV is right angled triangles with base 6cm and height 1.5cm
Area of region IV = $1/2 \times 6 \times 1.5 cm^2 = 4.5 cm^2$

So Total area of the paper used = $(2.488 + 6.5 + 1.3 + 4.5 + 4.5) cm^2 = 19.3\ cm^2$

Question 4

A triangle and a parallelogram have the same base and the same area. If the sides of the triangle are 26 cm, 28 cm and 30 cm, and the parallelogram stands on the base 28 cm, find the height of the parallelogram.

Answer

Given,
Area of the parallelogram and triangle are equal.

Length of the sides of the triangle are 26 cm, 28 cm and 30 cm.
Perimeter of the triangle = 26 + 28 + 30 = 84 cm
Semi perimeter of the triangle = 84/2 cm = 42 cm
Using heron's formula,
Area of the triangle = √s (s-a) (s-b) (s-c)
= √42(42 - 26) (46 - 28) (46 - 30) cm^2
= √46 × 16 × 14 × 16 cm^2
= 336 cm^2Let height of parallelogram be h.

Now given

Area of parallelogram = Area of triangle
Base × h = 336 cm^2

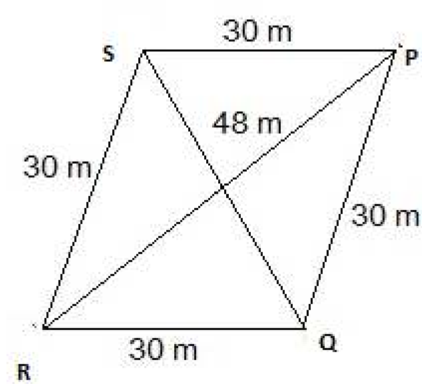
28cm × h = 336 cm^2
h = 336/28 cm
h = 12 cm
The height of the parallelogram is 12 cm.

Question 5

A rhombus shaped field has green grass for 18 cows to graze. If each side of the rhombus is 30 m and its longer diagonal is 48 m, how much area of grass field will each cow be getting?

Answer

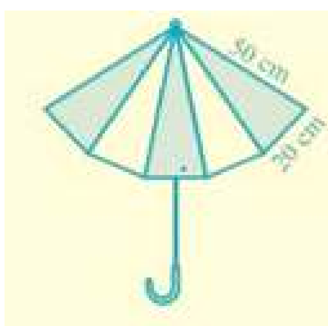
let PQRS be the rhombus shaped field and Diagonal QS divides the rhombus PQRS into two congruent triangles of equal area.



Semi perimeter of ΔPQS = (30 + 30 + 48)/2 m = 54 m
Using heron's formula,
Area of the ΔPQS = √s (s-a) (s-b) (s-c)
= √54(54 - 30) (54 - 30) (54 - 48) m^2
= √54 × 14 × 14 × 6 cm^2
= 432 m^2
Area of field = 2 × area of the ΔPQS = (2 × 432)m^2 = 864 m^2
So,
Area of grass field which each cow will be getting = 864/18 m^2 = 48 m^2

Question 6

An umbrella is made by stitching 10 triangular pieces of cloth of two different colors as shown below in figure. Each piece measuring 20 cm, 50 cm and 50 cm. How much cloth of each colour is required for the umbrella?



Answer

The total Umbrella area is equal 5 times area of one triangle.

Semi perimeter of each triangular piece = (50 + 50 + 20)/2 cm = 120/2 cm = 60cm

Using heron's formula,

Area of the triangular piece = $\sqrt{s (s-a) (s-b) (s-c)}$

$= \sqrt{60(60 - 50) (60 - 50) (60 - 20)}$ cm^2

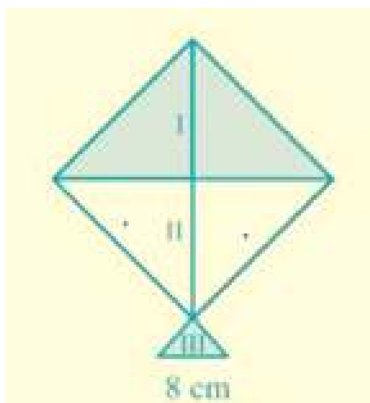
$= \sqrt{60 \times 10 \times 10 \times 40}$ cm^2

$= 200\sqrt{6}$ cm^2

Area of triangular piece = $5 \times 200\sqrt{6}$ cm^2 = $1000\sqrt{6}$ cm^2

Question 7

A kite in the shape of a square with a diagonal 32 cm and an isosceles triangle of base 8 cm and sides 6 cm each is to be made of three different shades as shown in below. How much paper of each shade has been used in it?



Answer

We know from the property of square the diagonals of a square bisect each other at right angle.

Also Area of given kite = 1/2 (diagonal)^2

$= 1/2 \times 32 \times 32 = 512$ cm^2

Area of shade I = Area of shade II

So it is equal to

= 512/2 cm^2 = 256 cm^2

So, area of paper required in each shade = 256 cm^2

For the III section, It is simple area calculation

Length of the sides of triangle = 6cm, 6cm and 8cm

Semi perimeter of triangle = (6 + 6 + 8)/2 cm = 10cm

Using heron's formula,

Area of the III triangular piece = √s (s-a) (s-b) (s-c)

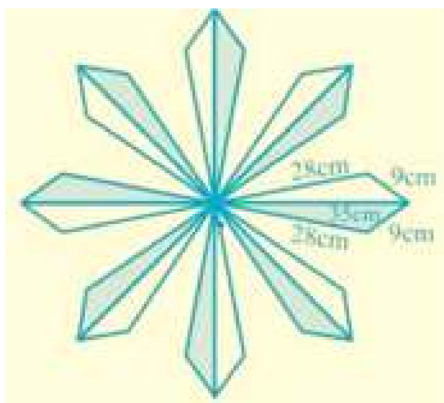
$= \sqrt{10(10 - 6)(10 - 6)(10 - 8)}\ cm^2$

$= \sqrt{10 \times 4 \times 4 \times 2}\ cm^2$

$= 8\sqrt{6}\ cm^2$

Question 8

A floral design on a floor is made up of 16 tiles which are triangular, the sides of the triangle being 9 cm, 28 cm and 35 cm . Find the cost of polishing the tiles at the rate of 50p per cm^2 .



Answer

Semi perimeter of each triangular shape = (28 + 9 + 35)/2 cm = 36 cm

Using heron's formula,

Area of each triangular shape = √s (s-a) (s-b) (s-c)

$= \sqrt{36(36 - 28)(36 - 9)(36 - 35)}\ cm^2$

$= \sqrt{36 \times 8 \times 27 \times 1}\ cm^2$

$= 36\sqrt{6}\ cm^2 = 88.2\ cm^2$

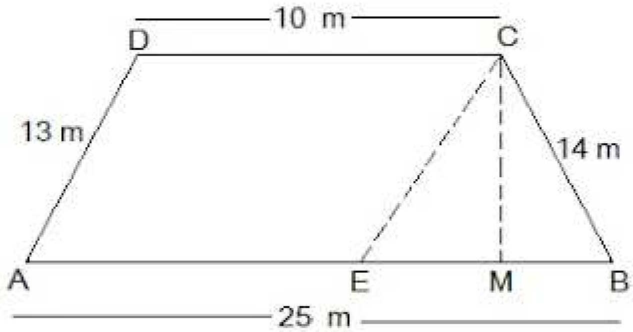
Total area of 16 tiles = 16 × 88.2 cm^2 = 1411.2 cm^2Cost of polishing tiles = 50p per cm^2

Total cost of polishing the tiles = Rs. (1411.2 × 0.5) = Rs. 705.6

Question 9

A field is in the shape of a trapezium whose parallel sides are 25 m and 10 m. The non-parallel sides are 14 m and 13 m. Find the area of the field.

Answer



Let ABCD be the given trapezium with parallel sides AB = 25m and CD = 10mand the non-parallel sides AD = 13m and BC = 14m.
We can drop a perpendicular from C on AM and draw a line CE parallel to DA

CM ⊥ AB and CE || AD.
This will divide the trapezium into two part
i) Parallelogram
ii) Triangle
Now
In ΔBCE,
BC = 14m, CE = AD = 13 m and
BE = AB - AE = 25 - 10 = 15m
Semi perimeter of the ΔBCE = (15 + 13 + 14)/2 m = 21 m
Using heron's formula,
Area of the ΔBCE = √s (s-a) (s-b) (s-c)
= √21(21 - 14) (21 - 13) (21 - 15) m^2
= √21 × 7 × 8 × 6 m^2
= 84 m^2
Now area of the ΔBCE = 1/2 × BE × CM = 84 m^2
1/2 × 15 × CM = 84 m^2
CM = 168/15 m^2
CM = 56/5 m^2
Area of the parallelogram AECD = Base × Altitude = AE × CM = 10 × 84/5 = 112 m^2
Area of the trapezium ABCD = Area of AECD + Area of ΔBCE
= (112+ 84) m^2 = 196 m^2