AP Exercise 1

Question 1

In which of the following situations, does the list of numbers involved make an arithmetic progression, and why?

(i) The taxi fare after each km when the fare is Rs 15 for the first km and Rs 8 for each additional km.

(ii) The amount of air present in a cylinder when a vacuum pump removes ¼ of the air remaining in the cylinder at a time.

(iii) The cost of digging a well after every meter of digging, when it costs Rs 150 for the first meter and rises by Rs 50 for each subsequent meter.

(iv) The amount of money in the account every year, when Rs 10000 is deposited at compound interest at 8 % per annum

Solution:

i) According to the question

Fare for First km= Rs 15

Fare for first km + additional 1 km = 15 +8

Fare for first km + additional 2 km = 15 +2X8

Fare for first km + additional 3 km = 15 +3X8

So series is like

15, 15 +8, 15 +2X8, 15 +3X8 …….

Difference between two terms =8 everywhere except first term

So it is Arithmetic Progression

ii) Let a be the amount of air initially

Amount of air remaining after 1^{st} pump =a –(a/4)=3a/4

Amount of air remaining after 2^{nd} pump= (3a/4) – (1/4)(3a/4)=9a/16

Amount of air remaining after 3^{rd} pump= (9a/16) – (1/4)(9a/16)=27a/64

So the series is like

a,3a/4,9a/4,27a/64……..

Difference Ist and second term=-a/4

Difference between Second and Third term= -3a/16

So difference is not constant

So it is not Arithmetic Progression

iii) According to the question

Cost of digging for First m= Rs 150

Cost of digging for First m + additional 1 m = 150 +50

Cost of digging for First m + additional 2 m = 150+2X50

Cost of digging for First m + additional 3 m = 150 +3X50

So series is like

150, 150 +50, 150 +2X50, 150 +3X50 …….

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Difference between two terms =50 everywhere except first term

So it is Arithmetic Progression

iv) According to the question

Money in account initially=10000

Money after 1st year =10000(1+.08)

Money after 2nd year=10000(1+.08)(1+.08)

So the series is

10000,10800,11664….

Difference between 2nd and Ist term=800

Difference between 3rd and 2nd term=864

As difference is not constant, it is not a AP

Question 2

Write first four terms of the AP, when the first term *a* and the common difference *d* are given as follows:

a = 10, *d* = 10

a = –2, *d* = 0

a = 4, *d* = – 3

a=-1 ,d=1/2

a = – 1.25, *d* = – 0.25

Solution:

Arithmetic Progression with first term a and common difference is shown

a,a+d,a+2d,a+3d……

Solving all these questions on the based of these formula

a=10,d=20

Series is 10,30,50,70

a=-2 ,d=0

Series is -2,-2,-2,-2

a=4,d=-3

Series is 4,1,-2,-5

a=-1,d=1/2

series is -1,-1/2,0,1/2

a=-1.25,d=-.25

series is -1.25,-1.50,-1.75,-2

Question 3

For the following APs, write the first term and the common difference:

(i) 3, 1, – 1, – 3, . . .

(ii) – 5, – 1, 3, 7, . . .

(iii) 1/3 , 5/3 , 9/3 , 13/3 ,…..

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(iv) 0.6, 1.7, 2.8, 3.9, . . .

Solution:

For any AP, First term is the number in the series and common difference is defined as difference of second term and first term

3, 1, – 1, – 3, . . .
First term=3
Common difference=1-3=-2
– 5, – 1, 3, 7, . . .
First term=-5
Common difference=-1-(-5)=4
1/3 , 5/3 , 9/3 , 13/3 ,.....
First term=1/3
Common difference=(5/3)-(1/3)=4/3
0.6, 1.7, 2.8, 3.9, . .
First term=.6
Common difference=1.7-.6=1.1

Question 4

Which of the following are APs? If they form an A.P. find the common difference d and write three more terms.

(i) 2, 4, 8, 16 …
(ii) 2, 5/2,3,7/2 ….
(iii) – 1.2, – 3.2, – 5.2, – 7.2 …
(iv) – 10, – 6, – 2, 2 …
(v) $3,3+\sqrt{2}\ ,\ 3+2\sqrt{2}\ ,3+3\sqrt{2}\ ,$….
(vi) 0.2, 0.22, 0.222, 0.2222 ….
(vii) 0, – 4, – 8, – 12 …
(viii) -1/2, -1/2,-1/2,-1/2….
(ix) 1, 3, 9, 27 …
(x) a, 2a, 3a, 4a …

(xi) a, a^2, a^3, a^4 …
(xii) $\sqrt{2}\ ,\sqrt{8}\ ,\sqrt{18}\ ,\sqrt{32}\ $….
(xiii) $\sqrt{3}\ ,\sqrt{6}\ ,\sqrt{9}\ ,\sqrt{12}\ $….
(xiv) 1^2, 3^2, 5^2, 7^2 …
(xv) 1^2, 5^2, 7^2, 73 …

Solution

For Arithmetic Progression, Common Difference should be same across
a,a+d,a+2d,a+3d
Let us assume four term given of series as
a_1 , a_2 , a_3 ,a_4
For the series to be AP,below should be true

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d= $a_2 - a_1 = a_3 - a_2 = a_4 - a_3$ ……(1)

If the series is AP ,then next term would

$a_5=a_4+d$

$a_6=a_4+2d$

$a_7=a_4+3d$

Now let us solves all the section as per theory given above

2, 4, 8, 16 …

So equation 1

2=4=8

This is not true, So it is not AP

2, 5/2,3,7/2 ….

So equation 1

D= 1/2=1/2 =1/2

This is True, So it is AP

$a_5=a_4+d$ = 7/2 + ½=4

$a_6=a_4+2d$= 7/2 +1=9/2

$a_7=a_4+3d$==5

− 1.2, − 3.2, − 5.2, − 7.2 …

So equation (1) is

D= -2=-2=-2

This is true, So it is AP

$a_5=a_4+d$ = -7.2 +(-2)=-9.2

$a_6=a_4+2d$= -7.2 +2(-2)=-11.2

$a_7=a_4+3d$==-7.2+ 3(-2)=-13.2

− 10, − 6, − 2, 2 …

So equation (1) is

d= 4=4=4

This is true, So it is AP

$a_5=a_4+d$ = 2 +(4)=6

$a_6=a_4+2d$= 2 +2(4)=10

$a_7=a_4+3d$==2+ 3(4)=14

$3, 3+\sqrt{2}$, $3+2\sqrt{2}$, $3+3\sqrt{2}$,….

So equation 1 becomes

D=$\sqrt{2}=\sqrt{2}=\sqrt{2}$

This is true ,So it is AP

$a_5=a_4+d$ = $3+4\sqrt{2}$

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$a_6=a_4+2d= 3+5\sqrt{2}$
$a_7=a_4+3d==3+6\sqrt{2}$

0.2, 0.22, 0.222, 0.2222 ….
So equation 1 becomes
D= .02=.002=.0003

Clearly this is not true, So it is not AP

vii) 0, – 4, – 8, – 12 …

So equation 1 becomes
D=-4=-4=-4
This is true, So it is AP
$a_5=a_4+d$ = -12 +(-4)=-16
$a_6=a_4+2d$= -12 +2(-4)=-20
$a_7=a_4+3d$==-12+ 3(-4)=-24

viii)) -1/2, -1/2,-1/2,-1/2….
Equation 1 becomes
D=0=0=0
So it is AP with zero Common difference
$a_5=a_4+d$ = -1/2
$a_6=a_4+2d$= -1/2
$a_7=a_4+3d$==-1/2
ix) 1, 3, 9, 27 …
Equation (1) becomes
D=2=6=18
Clearly not an AP

x) a, 2a, 3a, 4a …
Equation 1 becomes
D=a=a=a
Clearly an AP
$a_5=a_4+d$ = 5a
$a_6=a_4+2d$= 6a
$a_7=a_4+3d$==7a
xi) a, a^2, a^3, a^4 …
Equation 1 becomes
D=a^2 –a=a^3 - a^2 =a^4 – a^3
Clearly not an AP

xii) $\sqrt{2}$, $\sqrt{8}$, $\sqrt{18}$, $\sqrt{32}$ ….
It can rewritten as
$\sqrt{2}$, $2\sqrt{2}$, $3\sqrt{2}$, $4\sqrt{2}$ ….

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Equation 1 becomes
D=$\sqrt{2} = \sqrt{2} = \sqrt{2}$
This is true ,So it is AP
$a_5=a_4$ +d = $5\sqrt{2}$
$a_6=a_4$ +2d= $6\sqrt{2}$
$a_7=a_4$ +3d=$7\sqrt{2}$
xiii) $\sqrt{3}$, $\sqrt{6}$, $\sqrt{9}$, $\sqrt{12}$
equation 1 becomes
d=$\sqrt{6}$ - $\sqrt{3}$ =$\sqrt{9}$ - $\sqrt{6}$ = $\sqrt{12}$ - $\sqrt{9}$

Clearly not a AP

Xiv) 1^2, 3^2, 5^2, 7^2 …
Equation 1 becomes
D=8=16=24
Not an AP

xv) 1^2, 5^2, 7^2, 73 …
Equation (1) becomes
D=24=24=24
So an AP
$a_5=a_4$ +d = 97
$a_6=a_4$ +2d= 121
$a_7=a_4$ +3d==145

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