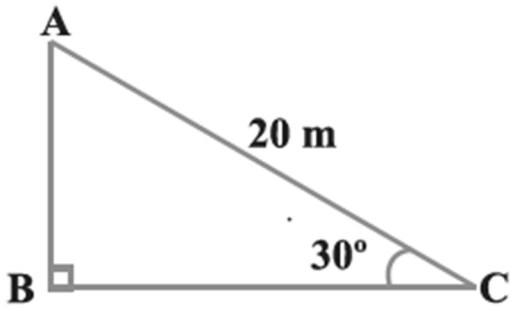
Applications of Trigonometry Exercise 1

Question 1

A circus artist is climbing a 20 m long rope, which is tightly stretched and tied from the top of a vertical pole to the ground. Find the height of the pole, if the angle made by the rope with the ground level is 30° (see below figure).



Solution

Let AB be the vertical pole AC be 20 m long rope tied to point C.
In ΔABC,

$\sin 30° = \text{Perpendicular/Base} = AB/AC$

$\Rightarrow 1/2 = AB/20$

$\Rightarrow AB = 20/2$

$\Rightarrow AB = 10$

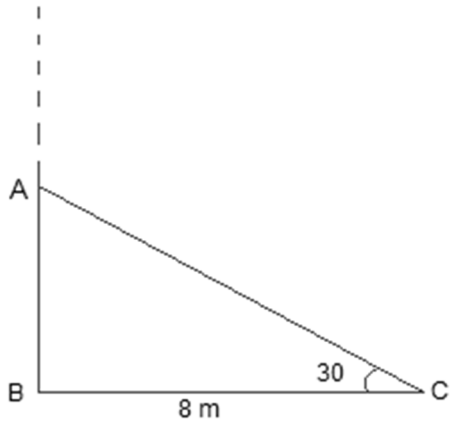
The height of the pole is 10 m.

Question 2

A tree breaks due to storm and the broken part bends so that the top of the tree touches the ground making an angle 30° with it. The distance between the foot of the tree to the point where the top touches the ground is 8 m. Find the height of the tree.

Solution

Let us draw the situation below



Let AC be the broken part of the tree.

AB +AC is the tree height

In right ΔABC,

cos 30° = Base/Hypotenuse =BC/AC

⇒ √3/2 = 8/AC

⇒ AC = 16/√3

Also,

tan 30° = AB/BC

⇒ 1/√3 = AB/8

⇒ AB = 8/√3

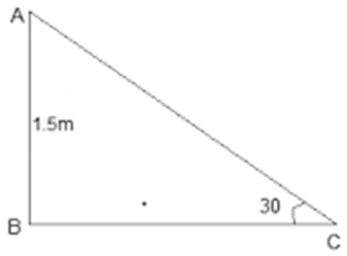
Total height of the tree = AB+AC = 16/√3 + 8/√3 = 24/√3

Question 3

A contractor plans to install two slides for the children to play in a park. For the children below the age of 5 years, she prefers to have a slide whose top is at a height of 1.5 m, and is inclined at an angle of 30° to the ground, whereas for elder children, she wants to have a steep slide at a height of 3 m, and inclined at an angle of 60° to the ground. What should be the length of the slide in each case?

Solution

Let us take small slide first



AB is the height and AC is the length of the slide

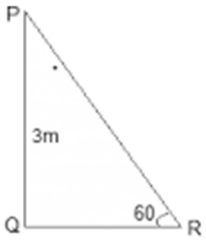
In right ΔABC,

sin 30° = Perpendicular/hypotenuse=AB/AC

⇒ 1/2 = 1.5/AC

⇒ AC = 3m

Now take the case of elder children slide



PQ is the height

PR is the length of slide

In r ΔPQR,

sin 60° = Perpendicular/hypotenuse= PQ/PR

⇒ √3/2 = 3/PR

⇒ PR = 2√3 m

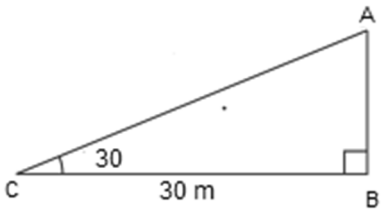
Hence, length of the slides are 3 m and 2√3 m respectively.

Question 4

The angle of elevation of the top of a tower from a point on the ground, which is 30 m away from the foot of the tower, is 30°. Find the height of the tower.

Solution:

The situation is depicted in the below figure



AB is height of the tower

BC =30 m (given)

In right ΔABC,

tan 30° = Perpendicular/base=AB/BC

⇒ 1/√3 = AB/30

⇒ AB = 10√3

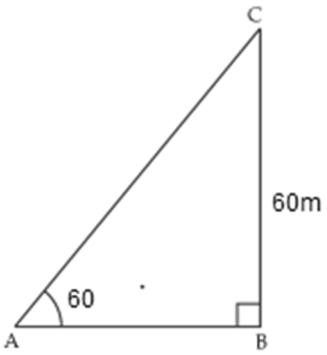
Thus, the height of the tower is 10√3 m.

Question 5

A kite is flying at a height of 60 m above the ground. The string attached to the kite is temporarily tied to a point on the ground. The inclination of the string with the ground is 60°. Find the length of the string, assuming that there is no slack in the string.

Solution:

The situation is depicted in the below figure



Let BC be the height of the kite from the ground,
AC be the inclined length of the string from the ground and A is the point where string of the kite is tied.

In ΔABC,

$\sin 60° = \text{Perpendicular/base} = BC/AC$

$\Rightarrow \sqrt{3}/2 = 60/AC$

$\Rightarrow AC = 40\sqrt{3}$ m

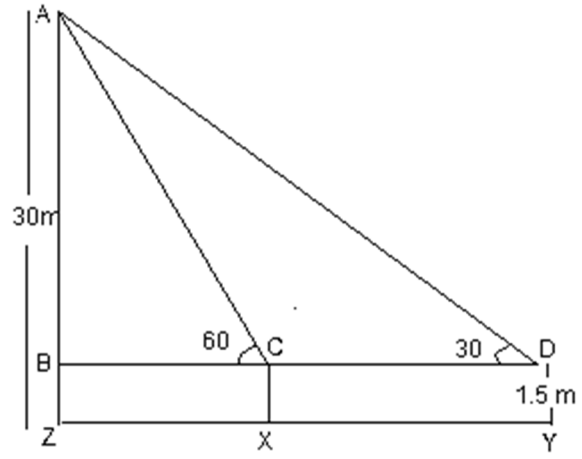
Thus, the length of the string from the ground is $40\sqrt{3}$ m.

Question 6

A 1.5 m tall boy is standing at some distance from a 30 m tall building. The angle of elevation from his eyes to the top of the building increases from 30° to 60° as he walks towards the building. Find the distance he walked towards the building

Solution:

The situation is depicted in the below figure



Given

DY=1.5m

Let the boy initially standing at point Y with inclination 30° and then he approaches the building to the point X with inclination 60°.

so XY is the distance he walked towards the building.

Height of the building = AZ=30 m

also, XY = CD.

AB = AZ - BZ = (30 - 1.5) = 28.5 m

In right ΔABD,

$\tan 30° = AB/BD$

$\Rightarrow 1/\sqrt{3} = 28.5/BD$

$\Rightarrow BD = 28.5\sqrt{3}$ m

also,

In right ΔABC,

$\tan 60° = AB/BC$

$\Rightarrow \sqrt{3} = 28.5/BC$

$\Rightarrow BC = 28.5/\sqrt{3} = 28.5\sqrt{3}/3$ m

$\therefore XY = CD = BD - BC = (28.5\sqrt{3} - 28.5\sqrt{3}/3) = 28.5\sqrt{3}(1-1/3) = 28.5\sqrt{3} \times 2/3 = 57/\sqrt{3}$ m.

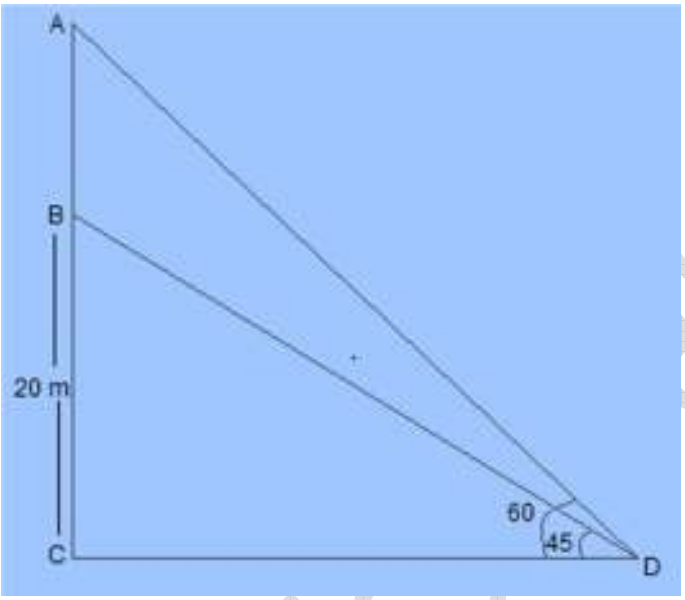
Thus, the distance boy walked towards the building is $57/\sqrt{3}$ m.

Question 7

From a point on the ground, the angles of elevation of the bottom and the top of a transmission tower fixed at the top of a 20 m high building are 45° and 60° respectively. Find the height of the tower.

Solution

The situation is depicted in the below figure



D is the point on Ground

BC be the 20 m high building..
Height of transmission tower = AB = AC - BC
In right ΔBCD,
$\tan 45° = BC/CD$
$\Rightarrow 1 = 20/CD$
$\Rightarrow CD = 20$ m
also,
In right ΔACD,
$\tan 60° = AC/CD$
$\Rightarrow \sqrt{3} = AC/20$

⇒ AC = 20√3 m

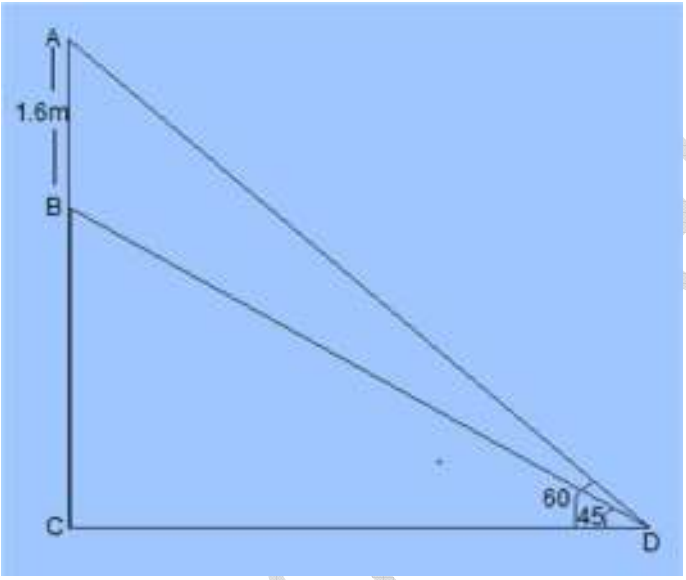
Height of transmission tower = AB = AC - BC = (20√3 - 20) m = 20(√3 - 1) m.

Question 8

A statue, 1.6 m tall, stands on the top of a pedestal. From a point on the ground, the angle of elevation of the top of the statue is 60° and from the same point the angle of elevation of the top of the pedestal is 45°. Find the height of the pedestal.

Solution:

The situation is depicted in the below figure



Let AB be the height of statue.

D is the point on the ground from where the elevation is taken.

Height of pedestal = BC = AC - AB

In right ΔBCD,

tan 45° = BC/CD

⇒ 1 = BC/CD

⇒ BC = CD.

also,

In right ΔACD,

$\tan 60° = AC/CD$

$\Rightarrow \sqrt{3} = AB+BC/CD$

$\Rightarrow \sqrt{3}CD = 1.6 \text{ m} + BC$

$\Rightarrow \sqrt{3}BC = 1.6 \text{ m} + BC$

$\Rightarrow \sqrt{3}BC - BC = 1.6 \text{ m}$

$\Rightarrow BC(\sqrt{3}-1) = 1.6 \text{ m}$

$\Rightarrow BC = 1.6/(\sqrt{3}-1) \text{ m}$

$\Rightarrow BC = 0.8(\sqrt{3}+1) \text{ m}$

Thus, the height of the pedestal is $0.8(\sqrt{3}+1)$ m.