Area Related to Circle Exercise 2

Things to Remember

$$\textbf{Area of the sector of angle } \theta = \frac{\theta}{360} \times \pi r^2$$

Length of the arc of angle θ = (θ/360) × 2πr

Unless stated otherwise, use π =22/7.

Question 1

Find the area of a sector of a circle with radius 6 cm if angle of the sector is 60°.

Solution

We know that

Area of the sector making angle θ = (θ/360°)×π r^2

Area of the sector making angle 60° = (60°/360°)×π r^2 cm²

    = (1/6)×6²π = 6 × 22/7 cm² = 132/7 cm²

Question 2

Find the area of a quadrant of a circle whose circumference is 22 cm.

Solution

We know that,

Circumference of the circle = 2πr = 22 cm

So, Radius of the circle = r = 22/2π cm = 7/2 cm

Now Quadrant of a circle means sector is making angle 90°.

Area of the sector making angle 90° = (90°/360°)×π r^2 cm²

    = (1/4)×(7/2)²π = (49/16) × (22/7) cm² = 77/8 cm²

Question 3

The length of the minute hand of a clock is 14 cm. Find the area swept by the minute hand in 5 minutes.

Solution

Here, Minute hand of clock acts as radius of the circle.
Therefore, Radius of the circle (r) = 14 cm
Angle rotated by minute hand in 1 hour = 360°
Therefore, Angle rotated by minute hand in 5 minutes = 360° × 5/60 = 30°
Area of the sector making angle 30° = (30°/360°)×π r^2 cm²
        = (1/12) × 14^2π = 196/12 π cm² = (49/3) × (22/7) cm² = 154/3 cm²
Area swept by the minute hand in 5 minutes = 154/3 cm²

Question 4

A chord of a circle of radius 10 cm subtends a right angle at the centre. Find the area of the corresponding:
(i) minor segment
(ii) major sector. (Use π = 3.14)

Solution
Given Radius of the circle = 10 cm
i) Height of ΔAOB = OA = 10 cm (radius of the circle)
Base of ΔAOB = OB = 10 cm (radius of the circle)
Area of ΔAOB = 1/2 × OA × OB [As Area = (1/2) Base X Height]
                = 50 cm²
Minor segment is making 90°
 So, Area of the sector making angle 90°
= (90°/360°) × π r^2 cm²
 = 25 × 3.14 cm² = 78.5 cm²
<u>Area of the minor segment</u> = Area of the sector making angle 90° - Area of ΔAOB
                = 78.5 cm² - 50 cm² = 28.5 cm²

Major segment is making 360° - 90° = 270°
Area of the sector making angle 270°
                = (270°/360°) × π r^2 cm²
                = 75 × 3.14 cm² = 235.5 cm²
Therefore, <u>Area of the major segment</u> = 235.5 cm²

Question 5

In a circle of radius 21 cm, an arc subtends an angle of 60° at the centre. Find:
  (i) the length of the arc

(ii) area of the sector formed by the arc

(iii) area of the segment formed by the corresponding chord

Solution

Given Radius of the circle = 21 cm

(i) As known from the Formula

Length of the arc = θ/360° × 2πr

$$= 60°/360° × 2 × 22/7 × 21$$
$$= 22 \text{ cm}$$

(ii) Angle subtend by the arc = 60°

Area of the sector making angle 60° = (60°/360°) × π r^2 cm²

$$= 441/6 \ \pi \text{ cm}^2$$
$$= 231 \text{ cm}^2$$

(iii) Area of equilateral ΔAOB = $\sqrt{3}/4$ × (OA)² = $\sqrt{3}/4$ × 21² = (441$\sqrt{3}$)/4 cm²

Area of the segment formed by the corresponding chord

= Area of the sector formed by the arc - Area of equilateral ΔAOB

= 231 cm² - (441$\sqrt{3}$)/4 cm²

Question 6

A chord of a circle of radius 15 cm subtends an angle of 60° at the centre. Find the areas of the corresponding minor and major segments of the circle. (Use π = 3.14 and $\sqrt{3}$ = 1.73)

Solution

The problem is depicted below

Major Segment

Minor Segment
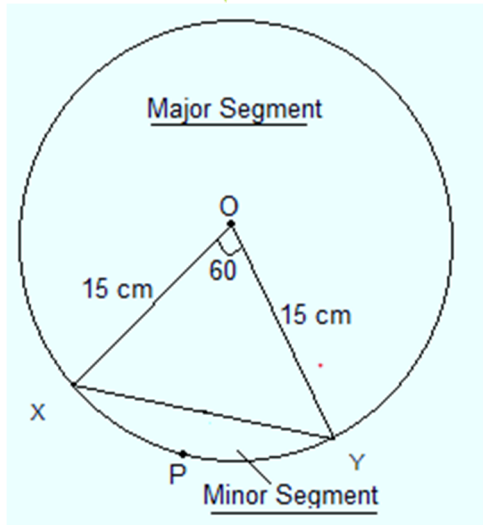
Radius of the circle = 15 cm

i) ΔXOY is isosceles as two sides are equal.

Therefore, ∠X = ∠Y

Sum of all angles of triangle = 180°

∠X + ∠O + ∠Y = 180°

2 ∠X = 180° - 60°

∠X = 60°

Therefore, Triangle is equilateral as ∠X = ∠Y = ∠O = 60°

Therefore, OX = OY = XY = 15 cm

Area of equilateral ΔXOY = $\sqrt{3}/4 \times (OX)^2 = \sqrt{3}/4 \times 15^2$

$= (225\sqrt{3})/4$ cm² = 97.3 cm²

Angle subtend at the centre by minor segment = 60°

Area of Minor sector making angle 60° = $(60°/360°) \times \pi r^2$ cm²

$= (1/6) \times 15^2 \pi$ cm² = 225/6 π cm²

$= (225/6) \times 3.14$ cm² = 117.75 cm²

Area of the minor segment = Area of Minor sector - Area of equilateral ΔXOY

= 117.75 - 97.3 = 20.4 cm²

Angle made by Major sector = 360° - 60° = 300°

Area of the sector making angle 300° = $(300°/360°) \times \pi r^2$ cm²

$= (5/6) \times 15^2 \pi = 1125/6 \pi$ cm²

= 588.75 cm²

Area of major segment = Area of Major sector + Area of equilateral ΔXOY
= 588.75 + 97.3 = 686.05 cm²

Question 7
A chord of a circle of radius 12 cm subtends an angle of 120° at the centre. Find the area of the corresponding segment of the circle. (Use π = 3.14 and √3 = 1.73)

Solution
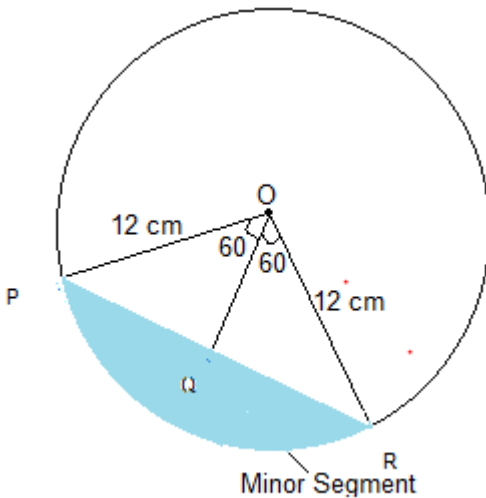
Radius of the circle, r = 12 cm
Draw a perpendicular OQ to chord PR. It will bisect PR.
In ΔOPQ
∠P = 180° - (90° + 60°) = 30°
cos 30° = PQ/OP
√3/2 = PQ/12
PQ = 6√3 cm
PR = 2 × PQ = 12√3 cm
sin 30° = OQ/OP
1/2 = OQ/12
OQ = 6 cm
Area of ΔPOR = 1/2 × base × height
= 1/2 × 12√3 × 6 = 36√3 cm
= 36 × 1.73 = 62.28 cm²
Angle made by Minor sector = 120°

Area of the sector making angle 120° = (120°/360°) × π r² cm²
= (1/3) × 12²π

= 150.72 cm²

Therefore, Area of the corresponding Minor segment = Area of the Minor sector - Area of ΔPOR   = 88.44 cm²

Question 8

A horse is tied to a peg at one corner of a square shaped grass field of side 15 m by means of a 5 m long rope .

Find
 (i) the area of that part of the field in which the horse can graze.
(ii) the increase in the grazing area if the rope were 10 m long instead of 5 m. (Use π = 3.14)

Solution

Side of square field = 15 m
Length of rope is the radius of the circle, r = 5 m
Since, the horse is tied at one end of square field, it will graze only quarter of the field with radius 5 m.

(i) Area of circle = πr^2 = 3.14 × 5² = 78.5 m²
Area of that part of the field in which the horse can graze = 1/4 of area of the circle = 78.5/4 = 19.625 m²

(ii) Area of circle if the length of rope is increased to 10 m = πr^2 = 3.14 × 10² = 314 m²
Area of that part of the field in which the horse can graze = 1/4 of area of the circle
                                 = 314/4 = 78.5 m²
Increase in grazing area = 78.5 m² - 19.625 m² = 58.875 m²

Question 9

A brooch is made with silver wire in the form of a circle with diameter 35 mm. The wire is also used in making 5 diameters which divide the circle into 10 equal sectors as shown below

 Find:
(i) the total length of the silver wire required.
(ii) the area of each sector of the brooch.

Solution

Number of diameters = 5
Length of diameter = 35 mm
Therefore, Radius = 35/2 mm
(i) Total length of silver wire required = Circumference of the circle + Length of 5 diameters

$$= 2\pi r + (5 \times 35) = (2 \times 22/7 \times 35/2) + 175$$
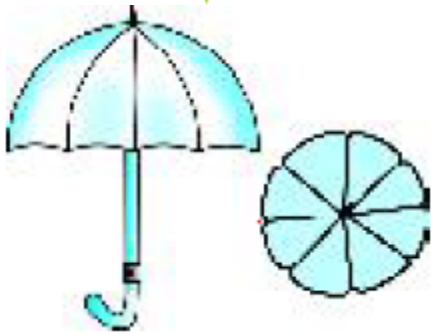$$= 110 + 175 \text{ mm} = 285 \text{ mm}$$

(ii) Number of sectors = 10
Area of each sector = Total area/Number of sectors
Total Area = πr^2 = 22/7 × (35/2)^2 = 1925/2 mm^2
Therefore, Area of each sector = (1925/2) × 1/10 = 385/4 mm^2

Question 10

An umbrella has 8 ribs which are equally spaced. Assuming umbrella to be a flat circle of radius 45 cm, find the area between the two consecutive ribs of the umbrella.

Solution

Number of ribs in umbrella = 8
Radius of umbrella while flat = 45 cm
Area between the two consecutive ribs of the umbrella =
                     Total area/Number of ribs
Total Area = πr^2 = 22/7 × (45)^2 = 6364.29 cm^2
Therefore, Area between the two consecutive ribs = 6364.29/8 cm^2
                                     = 795.5 cm^2

Question 11
A car has two wipers which do not overlap. Each wiper has a blade of length 25 cm sweeping through an angle of 115°. Find the total area cleaned at each sweep of the blades.

Solution

Angle of the sector of circle made by wiper = 115°
Radius of wiper = 25 cm
Area of the sector made by wiper = (115°/360°) × πr^2 cm^2
                 = 23/72 × 22/7 × 25^2 = 23/72 × 22/7 × 625 cm^2
                 = 158125/252 cm^2

Total area cleaned at each sweep of the blades = 2 ×158125/252 cm^2 = 158125/126
= 1254.96 cm^2

Question 12
To warn ships for underwater rocks, a lighthouse spreads a red coloured light over a sector of angle 80° to a distance of 16.5 km. Find the area of the sea over which the ships are warned.
(Use π = 3.14)

Solution

Let O bet the position of Lighthouse.
Distance over which light spread i.e. radius, r = 16.5 km
Angle made by the sector = 80°
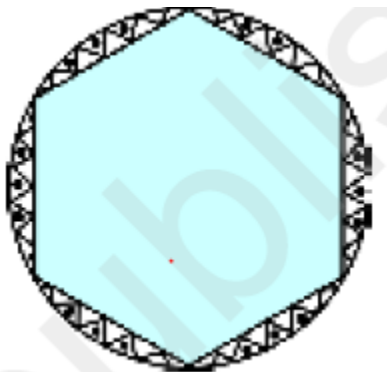Area of the sea over which the ships are warned = Area made by the sector.
Area of sector = (80°/360°) × π r^2
            = 2/9 × 3.14 × $(16.5)^2$
            = 189.97 km^2

Question 13
A round table cover has six equal designs as shown in below figure. If the radius of the cover is 28 cm, find the cost of making the designs at the rate of ₹ 0.35 per cm^2 .
(Use √3 = 1.7)
Solution

Number of equal designs = 6
Radius of round table cover = 28 cm
Cost of making design = ₹ 0.35 per cm^2
∠O = 360°/6 = 60°

ΔAOB is isosceles as two sides are equal. (Radius of the circle)
Therefore, ∠A = ∠B
Sum of all angles of triangle = 180°
∠A + ∠B + ∠O = 180°
⇒ 2 ∠A = 180° - 60°
⇒ ∠A = 120°/2

⇒ ∠A = 60°

Triangle is equilateral as ∠A = ∠B = ∠C = 60°

Area of equilateral ΔAOB = $\sqrt{3}/4 \times (OA)^2 = \sqrt{3}/4 \times 28^2 = 333.2$ cm^2

Area of sector ACB = $(60°/360°) \times \pi r^2$ cm^2

      $= 1/6 \times 22/7 \times 28 \times 28 = 410.66$ cm^2

Area of design = Area of sector ACB - Area of equilateral ΔAOB

      $= 410.66$ cm^2 - 333.2 cm^2 = 77.46 cm^2

Area of 6 design = 6×77.46 cm^2 = 464.76 cm^2

Total cost of making design = 464.76 cm$^2 \times$ ₹ 0.35 per cm^2 = ₹ 162.66

Question 14

Tick the correct Solution in the following:

Area of a sector of angle p (in degrees) of a circle with radius R is

  (A) $p/180 \times 2\pi R$

  (B) $p/180 \times \pi R^2$

  (C) $p/360 \times 2\pi R$

  (D) $p/720 \times 2\pi R^2$

Solution

Area of a sector of angle p = $p/360 \times \pi R^2$

       $= p/360 \times 2/2 \times \pi R^2$

       $= 2p/720 \times 2\pi R^2$

Hence, Option (D) is correct.