

Coordinate Geometry Exercise 1

Question 1 Find the distance between the following pairs of points

- (i) $(2, 3), (4, 1)$
- (ii) $(-5, 7), (-1, 3)$
- (iii) $(a, b), (-a, -b)$

Question 2 Find the distance between the points $(0, 0)$ and $(36, 15)$. Can you now find the distance between the two towns A and B?

Question 3 Determine if the points $(1, 5), (2, 3)$ and $(-2, -11)$ are collinear.

Question 4 Check whether $(5, -2), (6, 4)$ and $(7, -2)$ are the vertices of an isosceles triangle

Question 5 In a classroom, 4 friends are seated at the points A, B, C and D as shown below. Champa and Chameli walks into the class and after observing for a few minutes Champa asks Chameli, "Don't you think ABCD is a square?" Chameli disagrees. Using distance formula, find Which of them is correct?

Question 6 Name the type of quadrilateral formed, if any, by the following points, and give reasons for your answer:

- (i) $(-1, -2), (1, 0), (-1, 2), (-3, 0)$
- (ii) $(-3, 5), (3, 1), (0, 3), (-1, -4)$
- iii) $(4,5), (7,6), (4,3), (1,2)$

Question 7 Find the point on the x -axis which is equidistant from $(2, -5)$ and $(-2, 9)$.

Question 8 Find the values of y for which the distance between the points $P(2, -3)$ and $Q(10, y)$ is 10 units

Question 9 If $Q(0, 1)$ is equidistant from $P(5, -3)$ and $R(x, 6)$, find the values of x . Also find the distances QR and PR .

Question 10 Find a relation between x and y such that the point (x, y) is equidistant from the point $(3, 6)$ and $(-3, 4)$.

Solution 1

Distance between the points AB is given by

$$D = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

- i) $D = \sqrt{(4 - 2)^2 + (1 - 3)^2} = 2\sqrt{2}$
- ii) $D = \sqrt{(-1 + 5)^2 + (3 - 7)^2} = 4\sqrt{2}$
- iii) $D = \sqrt{(-a - a)^2 + (-b - b)^2} = 2\sqrt{a^2 + b^2}$

Solution 2

Let P(0,0) and Q(36,15) be the given points.

Distance between the points PQ is given by

$$D = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$D = \sqrt{(36 - 0)^2 + (15 - 0)^2} = 39$$

The position of town A and B can be represented as points P and Q respectively, so distance between towns will be 39 km

Solution3

Let's us denote point by P(1,5) , Q(2,3) and R(-2,-11)

If the points are not collinear, then we should be able to form the triangle

Let's us find the length of PQ, QR and PR by distance formula

$$D = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$PQ = \sqrt{(2 - 1)^2 + (3 - 5)^2} = \sqrt{5}$$

$$QR = \sqrt{(-2 - 2)^2 + (-11 - 3)^2} = \sqrt{212}$$

$$PR = \sqrt{(-2 - 1)^2 + (-11 - 5)^2} = \sqrt{265}$$

Clearly none of these is true

$$PQ + QR = PR$$

$$PR + PQ = QR$$

$$PQ = QR + PR$$

Hence they are not collinear

Solution 4

Let us denote point by P(5,-2) , Q(6,4) and R(7,-2)

Let us find the length of PQ, QR and PR by distance formula

$$D = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$PQ = \sqrt{(6 - 5)^2 + (4 + 2)^2} = \sqrt{37}$$

$$QR = \sqrt{(7 - 6)^2 + (-2 - 4)^2} = \sqrt{37}$$

$$PR = \sqrt{(7 - 5)^2 + (-2 + 2)^2} = 2$$

Now PQ = QR, so it is an isosceles triangle

Solution 5

As per the figure given, The coordinates of the points A,B,C and D are (3,4), (6,7), (9,4) and (6,1)

Let us find the length of AB, BC, CD and AD by distance formula

$$D = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$AB = \sqrt{(6 - 3)^2 + (7 - 4)^2} = 3\sqrt{2}$$

$$BC = \sqrt{(9 - 6)^2 + (4 - 7)^2} = 3\sqrt{2}$$

$$CD = \sqrt{(6 - 9)^2 + (1 - 4)^2} = 3\sqrt{2}$$

$$DA = \sqrt{(3 - 6)^2 + (4 - 1)^2} = 3\sqrt{2}$$

So all the sides are equal. But we cannot still say that it is square as rhombus has all the sides equal also.

Now we know that a square has both the diagonal equal also, So lets us calculate the diagonal's

$$AC = \sqrt{(9 - 3)^2 + (4 - 4)^2} = 6$$

$$BD = \sqrt{(6 - 6)^2 + (1 - 7)^2} = 6$$

Hence AC=BD

So it is a square.

So champa is correct

Solution 6

In this type of problem, we need to find of length of each segment, then check with the properties of different type of quadrilateral

i.e. for points A,B,C and D

Line segments are AB, BC,AC,CD,AD and BD

We need to find the length of each of these

i) $(-1, -2), (1, 0), (-1, 2), (-3, 0)$

$$D = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$AB = \sqrt{(1 + 1)^2 + (0 + 2)^2} = 2\sqrt{2}$$

$$BC = \sqrt{(-1 - 1)^2 + (2 - 0)^2} = 2\sqrt{2}$$

$$CD = \sqrt{(-3 + 1)^2 + (0 - 2)^2} = 2\sqrt{2}$$

$$DA = \sqrt{(-1 + 3)^2 + (-2 - 0)^2} = 2\sqrt{2}$$

$$AC = \sqrt{(-1 + 1)^2 + (2 + 2)^2} = 4$$

$$BD = \sqrt{(-3 - 1)^2 + (0 - 0)^2} = 4$$

Here the side AB, BC, CD and AD are equal and diagonal AC and BD are equal. So this is a square

ii) $(-3, 5), (3, 1), (0, 3), (-1, -4)$

$$D = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$AB = \sqrt{(3 + 3)^2 + (1 - 5)^2} = 2\sqrt{13}$$

$$BC = \sqrt{(0 - 3)^2 + (3 - 1)^2} = \sqrt{13}$$

$$CD = \sqrt{(-1 - 0)^2 + (-4 - 3)^2} = \sqrt{50}$$

$$DA = \sqrt{(-1 + 3)^2 + (-4 - 5)^2} = \sqrt{85}$$

$$AC = \sqrt{(0 + 3)^2 + (3 - 5)^2} = \sqrt{13}$$

$$BD = \sqrt{(-1 - 3)^2 + (-4 - 1)^2} = \sqrt{41}$$

Now here $AC+BC=AB$

So that means ABC are collinear points.

So it is not a quadrilateral infact

iii) (4,5) ,(7,6), (4,3) (1,2)

$$D = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$AB = \sqrt{(7 - 4)^2 + (6 - 5)^2} = \sqrt{10}$$

$$BC = \sqrt{(4 - 7)^2 + (3 - 6)^2} = \sqrt{18}$$

$$CD = \sqrt{(1 - 4)^2 + (2 - 3)^2} = \sqrt{10}$$

$$DA = \sqrt{(4 - 1)^2 + (5 - 2)^2} = \sqrt{18}$$

$$AC = \sqrt{(4 - 4)^2 + (3 - 5)^2} = 2$$

$$BD = \sqrt{(1 - 7)^2 + (2 - 6)^2} = \sqrt{52}$$

Now $AB=CD$ and $BC=DA$

Now it could be rectangle or parallelogram

But diagonal AC is not equal diagonal BD

So It is a parallelogram

Solution 7

Since the point lies on X axis, the point should be of the form (a,0)

Now (a,0) is equidistant from both the given points

$$\sqrt{(x-2)^2 + (0+5)^2} = \sqrt{(x+2)^2 + (0-9)^2}$$

Squaring both the sides

$$(x-2)^2 + (0+5)^2 = (x+2)^2 + (0-9)^2$$

Solving it we get

$$x = -7$$

Solution 8

According to the question

$$PQ = 10$$

$$10 = \sqrt{(10-2)^2 + (y+3)^2}$$

Or

$$10 = \sqrt{y^2 + 6y + 73}$$

Squaring both the sides

$$y^2 + 6y - 27 = 0$$

$$(y+9)(y-3) = 0$$

So $y = -9$ or 3

Solution 9

Now

$$QP=QR$$

$$\sqrt{(5-0)^2 + (-3-1)^2} = \sqrt{(x-0)^2 + (6-1)^2}$$

Squaring both the sides

$$25+16=x^2 +25$$

$$x= -4 \text{ or } 4$$

So point R is either (4,6) or (-4,6)

Distance QR and PR when R (4,6)

$$QR = \sqrt{(4-0)^2 + (6-1)^2} = \sqrt{41}$$

$$PR = \sqrt{(4-5)^2 + (6+3)^2} = \sqrt{82}$$

Distance QR and PR when R (-4,6)

$$QR = \sqrt{(-4-0)^2 + (6-1)^2} = \sqrt{41}$$

$$PR = \sqrt{(-4-5)^2 + (6+3)^2} = 9\sqrt{2}$$

Solution 10

Let the point P(x,y) is equidistant from the point Q(3,6) and R(-3,4)

$$PQ=PR$$

$$\sqrt{(x-3)^2 + (y-6)^2} = \sqrt{(x+3)^2 + (y-4)^2}$$

Squaring both the sides

$$x^2 -6x+9 +y^2-12y +36=x^2 +6x+9 +y^2 -8y+16$$

$$-12x-4y+20=0$$

Or

$$3x+y-5=0 \text{ (dividing by -4)}$$

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