

Quadratic Equations Exercise -2

We can solve Quadratic equation using Factorization

S.no	Method	Working
1	factorization	<p>This method we factorize the equation by splitting the middle term b</p> <p>In $ax^2+bx+c=0$</p> <p>Example</p> <p>$6x^2-x-2=0$</p> <p>1) First we need to multiple the coefficient a and c. In this case $=6 \times -2 = -12$</p> <p>2) Splitting the middle term so that multiplication is 12 and difference is the coefficient b</p> <p>$6x^2 + 3x - 4x - 2 = 0$</p> <p>$3x(2x+1) - 2(2x+1) = 0$</p> <p>$(3x-2)(2x+1) = 0$</p> <p>3) Roots of the equation can be find equating the factors to zero</p> <p>$3x-2=0 \Rightarrow x=2/3$</p> <p>$2x+1=0 \Rightarrow x=-1/2$</p>

Question 1.

Find the roots of the following quadratic equations by factorization:

(i) $x^2 - 3x - 10 = 0$

(ii) $2x^2 + x - 6 = 0$

(iii) $\sqrt{2}x^2 + 7x + 5\sqrt{2} = 0$

(iv) $2x^2 - x + 1/8 = 0$

$$(v) 100x^2 - 20x + 1 = 0$$

Answer

$$\begin{aligned} (i) x^2 - 3x - 10 &= 0 \\ x^2 - 5x + 2x - 10 &= 0 \\ x(x - 5) + 2(x - 5) &= 0 \\ (x - 5)(x + 2) &= 0 \\ \therefore x - 5 = 0 \text{ or } x + 2 = 0 \\ x = 5 \text{ or } x = -2 \end{aligned}$$

$$\begin{aligned} (ii) 2x^2 + x - 6 &= 0 \\ 2x^2 + 4x - 3x - 6 &= 0 \\ 2x(x + 2) - 3(x + 2) &= 0 \\ (x + 2)(2x - 3) &= 0 \\ \therefore x + 2 = 0 \text{ or } 2x - 3 = 0 \\ x = -2 \text{ or } x = 3/2 \end{aligned}$$

$$\begin{aligned} (iii) \sqrt{2} x^2 + 7x + 5\sqrt{2} &= 0 \\ \sqrt{2} x^2 + 5x + 2x + 5\sqrt{2} &= 0 \\ x(\sqrt{2}x + 5) + \sqrt{2}(\sqrt{2}x + 5) &= 0 \\ (\sqrt{2}x + 5)(x + \sqrt{2}) &= 0 \\ \therefore \sqrt{2}x + 5 = 0 \text{ or } x + \sqrt{2} = 0 \\ x = -5/\sqrt{2} \text{ or } x = -\sqrt{2} \end{aligned}$$

$$\begin{aligned} (iv) 2x^2 - x + 1/8 &= 0 \\ 1/8 (16x^2 - 8x + 1) &= 0 \\ 1/8 (16x^2 - 4x - 4x + 1) &= 0 \\ 1/8 (4x(4x - 1) - 1(4x - 1)) &= 0 \\ 1/8(4x - 1)^2 &= 0 \\ (4x - 1) = 0 \text{ or } (4x - 1) = 0 \\ x = 1/4 \text{ or } x = 1/4 \end{aligned}$$

$$\begin{aligned} (v) 100x^2 - 20x + 1 &= 0 \\ 100x^2 - 10x - 10x + 1 &= 0 \\ 10x(10x - 1) - 1(10x - 1) &= 0 \\ (10x - 1)^2 &= 0 \\ \therefore (10x - 1) = 0 \text{ or } (10x - 1) = 0 \\ x = 1/10 \text{ or } x = 1/10 \end{aligned}$$

Question 2.

(i) John and Jivanti together have 45 marbles. Both of them lost 5 marbles each, and the product of the number of marbles they now have is 124. Find out how many marbles they had to start with.

(ii) A cottage industry produces a certain number of toys in a day. The cost of production of each toy (in rupees) was found to be 55 minus the number of toys produced in a day. On a particular day, the total cost of production was Rs 750. Find out the number of toys produced on that day.

Answer

i) Let the number of John's marbles be x .
Therefore, number of Jivanti's marble = $45 - x$
After losing 5 marbles,
Number of John's marbles = $x - 5$
Number of Jivanti's marbles = $45 - x - 5 = 40 - x$
As per question the product of their marbles is 124.

$$\therefore (x - 5)(40 - x) = 124$$

$$x^2 - 45x + 324 = 0$$

Solving by factorization method

$$x^2 - 36x - 9x + 324 = 0$$

$$x(x - 36) - 9(x - 36) = 0$$

$$(x - 36)(x - 9) = 0$$

Either $x - 36 = 0$ or $x - 9 = 0$

$$\Rightarrow x = 36 \text{ or } x = 9$$

If the number of John's marbles = 36,

Then, number of Jivanti's marbles = $45 - 36 = 9$

If number of John's marbles = 9,

Then, number of Jivanti's marbles = $45 - 9 = 36$

ii)

Let the number of toys produced be x .

\therefore Cost of production of each toy = Rs $(55 - x)$

As per problem, total production of the toys = Rs 750

$$\therefore x(55 - x) = 750$$

$$x^2 - 55x + 750 = 0$$

Solving by factorization method

$$x^2 - 25x - 30x + 750 = 0$$

$$x(x - 25) - 30(x - 25) = 0$$

$$(x - 25)(x - 30) = 0$$

Either, $x - 25 = 0$ or $x - 30 = 0$

$$\Rightarrow x = 25 \text{ or } x = 30$$

Hence, the number of toys will be either 25 or 30.

Question 3

Find two numbers whose sum is 27 and product is 182.

Answer

Let the first number be x and the second number is $27 - x$.

Therefore, their product = $x(27 - x)$

Now as per question the product of these numbers is 182.

So, $x(27 - x) = 182$

$$x^2 - 27x + 182 = 0$$

$$x^2 - 13x - 14x + 182 = 0$$

$$x(x - 13) - 14(x - 13) = 0$$

$$(x - 13)(x - 14) = 0$$

Either $x - 13 = 0$ or $x - 14 = 0$

$\Rightarrow x = 13$ or $x = 14$

If first number = 13, then

Other number = $27 - 13 = 14$

If first number = 14, then

Other number = $27 - 14 = 13$

Therefore, the numbers are 13 and 14.

Question 4

Find two consecutive positive integers, sum of whose squares is 365.

Answer

Let the consecutive positive integers be x and $x + 1$.

Therefore, $x^2 + (x + 1)^2 = 365$

$$x^2 + x^2 + 1 + 2x = 365$$

$$2x^2 + 2x - 364 = 0$$

$$x^2 + x - 182 = 0$$

Solving by factorization method

$$x^2 + 14x - 13x - 182 = 0$$

$$x(x + 14) - 13(x + 14) = 0$$

$$(x + 14)(x - 13) = 0$$

Either $x + 14 = 0$ or $x - 13 = 0$,

$\Rightarrow x = -14$ or $x = 13$

Since the integers are positive, x can only be 13.

$\therefore x + 1 = 13 + 1 = 14$

Therefore, two consecutive positive integers will be 13 and 14.

Question 5

The altitude of a right triangle is 7 cm less than its base. If the hypotenuse is 13 cm, find the other two sides.

Answer

Let the base of the right triangle be x cm.

Its altitude = $(x - 7)$ cm

From Pythagoras theorem, we have

$$\text{Base}^2 + \text{Altitude}^2 = \text{Hypotenuse}^2$$

$$\therefore x^2 + (x - 7)^2 = 132$$

$$x^2 + x^2 + 49 - 14x = 169$$

$$2x^2 - 14x - 120 = 0$$

$$x^2 - 7x - 60 = 0$$

Solving by factorization method

$$x^2 - 12x + 5x - 60 = 0$$

$$x(x - 12) + 5(x - 12) = 0$$

$$(x - 12)(x + 5) = 0$$

Either $x - 12 = 0$ or $x + 5 = 0$,

$$\Rightarrow x = 12 \text{ or } x = -5$$

Since sides are positive, x can only be 12.

Therefore, the base of the given triangle is 12 cm and the altitude of this triangle will be $(12 - 7)$ cm = 5 cm.

Question 6

A cottage industry produces a certain number of pottery articles in a day. It was observed on a particular day that the cost of production of each article (in rupees) was 3 more than twice the number of articles produced on that day. If the total cost of production on that day was Rs 90, find the number of articles produced and the cost of each article.

Answer

Let the number of articles produced be x .

Therefore, cost of production of each article = Rs $(2x + 3)$

As per question that the total production is Rs 90

$$\text{So } x(2x + 3) = 90$$

$$2x^2 + 3x - 90 = 0$$

Solving by factorization method

$$2x^2 + 15x - 12x - 90 = 0$$

$$x(2x + 15) - 6(2x + 15) = 0$$

$$(2x + 15)(x - 6) = 0$$

Either $2x + 15 = 0$ or $x - 6 = 0$

$$\Rightarrow x = -15/2 \text{ or } x = 6$$

As the number of articles produced can only be a positive integer, therefore, x can only be 6.

Hence, number of articles produced = 6

Cost of each article = $2 \times 6 + 3 = \text{Rs } 15$.