

Trigonometry Exercise 1

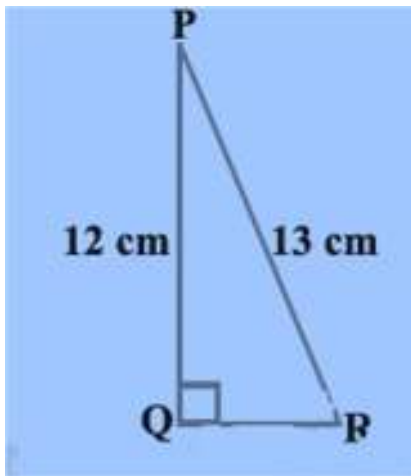
Question 1

In $\triangle ABC$, right-angled at B, $AB = 24$ cm, $BC = 7$ cm. Determine:

- (i) $\sin A$, $\cos A$
- (ii) $\sin C$, $\cos C$

Question 2

In below find $\tan P - \cot R$



Question 3

If $\sin A = 3/4$

Calculate $\cos A$ and $\tan A$.

Question 4

Given $15 \cot A = 8$, find $\sin A$ and $\sec A$.

Question 5

Given $\sec \theta = 13/12$

Calculate all other trigonometric ratios.

Question 6

If $\angle A$ and $\angle B$ are acute angles such that $\cos A = \cos B$, then show that $\angle A = \angle B$.

Question 7

If $\cot \theta = 7/8$

evaluate :

- (i) $\frac{(1+\sin\theta)(1-\sin\theta)}{(1+\cos\theta)(1-\cos\theta)}$
 (ii) $\cot^2 \theta$

Question 8

If $3 \cot A = 4$, check whether below is true or not

$$\frac{1 - \tan^2 A}{1 + \tan^2 A} = \cos^2 A - \sin^2 A$$

Question 9

In triangle ABC, right-angled at B, if $\tan A = \frac{1}{\sqrt{3}}$

Find the value of:

- (i) $\sin A \cos C + \cos A \sin C$
 (ii) $\cos A \cos C - \sin A \sin C$

Question 10

In ΔPQR , right-angled at Q, $PR + QR = 25$ cm and $PQ = 5$ cm. Determine the values of $\sin P$, $\cos P$ and $\tan P$.

Question 11

State whether the following are true or false. Justify your answer.

- (i) The value of $\tan A$ is always less than 1.
 (ii) $\sec A = 12/5$ for some value of angle A.
 (iii) $\cos A$ is the abbreviation used for the cosecant of angle A.
 (iv) $\cot A$ is the product of \cot and A.
 (v) $\sin \theta = 4/3$ for some angle

Solution 1

In ΔABC , right-angled at B, using Pythagoras theorem we have

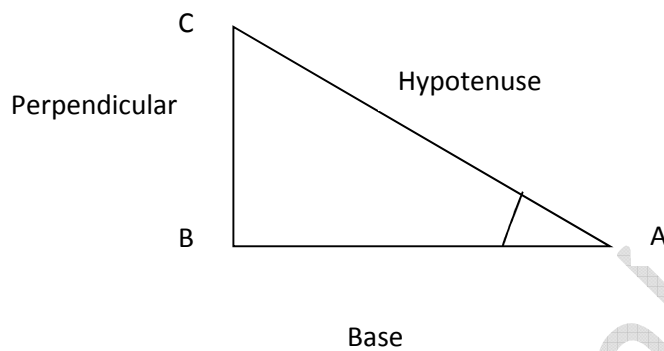
$$AC^2 = AB^2 + BC^2$$

$$= 576 + 49 = 625$$

Or $AC = 25$ (taking positive value only)

Now

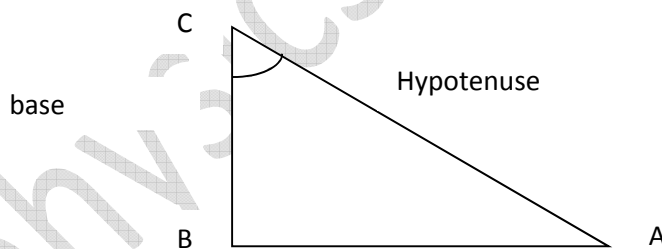
In a right angle triangle ABC where $B = 90^\circ$,



$$\sin A = BC/AC = 7/25$$

$$\cos A = AB/AC = 24/25$$

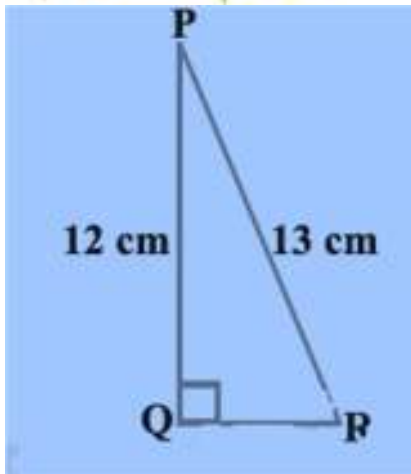
ii)



$$\sin C = AB/AC = 24/25$$

$$\cos C = BC/AC = 7/25$$

Solution 2



Now by Pythagoras theorem
 $PQ^2 + QR^2 = PR^2$
 $QR = 5$

Now
 $\tan P = \frac{\text{Perpendicular}}{\text{Base}} = \frac{5}{12}$
 $\cot R = \frac{\text{Base}}{\text{Perpendicular}} = \frac{5}{12}$

So $\tan P - \cot R = 0$

Solution 3

Given $\sin A = \frac{3}{4}$
 Or $\frac{P}{H} = \frac{3}{4}$
 Let $P = 3k$ and $H = 4k$



Now By Pythagoras theorem
 $P^2 + B^2 = H^2$
 $9k^2 + B^2 = 16k^2$
 Or $B = +k\sqrt{7}$

Now $\cos A = \frac{B}{H} = \frac{\sqrt{7}}{4}$

Now $\tan A = \frac{\sin A}{\cos A} = \frac{3}{\sqrt{7}}$

Solution 4

$$\cot A = 8/15$$

Or

$$B/P = 8/15$$

$$\text{Let } B = 8K \text{ and } P = 15k$$



So in a right angle triangle with angle A

$$P^2 + B^2 = H^2$$

$$\text{Or } H = 17K$$

$$\sin A = P/H = 15/17$$

$$\sec A = H/B = 17/8$$

Solution 5

$$\text{Given } \sec \theta = 13/12$$

Or

$$H/B = 13/12$$

$$\text{let } H = 13K, B = 12K$$



So in a right angle triangle with angle A

$$P^2 + B^2 = H^2$$

$$P = 5k$$

$$\sin \theta = P/H = 5/13$$

$$\cos \theta = B/H = 12/13$$

$$\tan \theta = P/B = 5/12$$

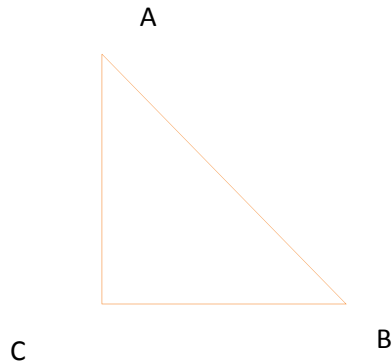
$$\operatorname{cosec} \theta = 1/\sin \theta = 13/5$$

$$\cot \theta = 1/\tan \theta = 12/5$$

Solution 6

In a triangle

$$\cos A = \cos B$$



$$AC/AB = BC/AB$$

$$\Rightarrow AC = BC$$

$$\Rightarrow \text{Angle A and Angle B}$$

Solution 7

Given

$$\text{Given } \cot \theta = 7/8$$

Or

$$B/P = 7/8$$

$$\text{let } B = 7K, P = 8K$$



So in a right angle triangle with angle θ

$$P^2 + B^2 = H^2$$

$$H = k\sqrt{113}$$

$$\sin \theta = P/H = \frac{8}{\sqrt{113}}$$

$$\cos \theta = B/H = \frac{7}{\sqrt{113}}$$

$$(i) \frac{(1+\sin\theta)(1-\sin\theta)}{(1+\cos\theta)(1-\cos\theta)}$$

$$= \frac{1-\sin^2\theta}{1-\cos^2\theta}$$

$$= \frac{1-\frac{64}{113}}{1-\frac{49}{113}} = 49/64$$

$$i) \cot^2 \theta = (\cot \theta)^2 = 49/64$$

Solution 8

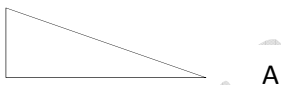
Given

$$\cot A = 4/3$$

Or

$$B/P = 4/3$$

Let $B=4k$ and $P=3k$



So in a right angle triangle with angle A

$$P^2 + B^2 = H^2$$

$$H=5k$$

$$\text{Now } \tan A = 1/\cot A = 3/4$$

$$\cos A = B/H = 4/5$$

$$\sin A = P/H = 3/5$$

Let us take the LHS

$$\frac{1 - \tan^2 A}{1 + \tan^2 A}$$

$$= \frac{1 - \left(\frac{3}{4}\right)^2}{1 + \left(\frac{3}{4}\right)^2} = \frac{7}{25}$$

$$\text{RHS} = \cos^2 A - \sin^2 A = 7/25$$

So LHS=RHS, so the statement is true

Solution 9

$$\tan A = \frac{1}{\sqrt{3}}$$

$$P/B = \frac{1}{\sqrt{3}}$$

Let

$$P = k \text{ and } B = k\sqrt{3}$$

Now by Pythagoras theorem

$$P^2 + B^2 = H^2$$

$$H = 2k$$

- i) $\sin A \cos C + \cos A \sin C = (P/H)(B/H) + (B/H)(P/H)$
 $= (BC/AC)(BC/AC) + (AB/AC)(AB/AC)$
 $= k^2/4k^2 + 3k^2/4k^2 = 1$
- ii) $\cos A \cos C - \sin A \sin C$
 $= (P/H)(P/H) - (B/H)(B/H)$
 $= (BC/AC)(AB/AC) - (AB/AC)(BC/AC)$
 $= 0$

Solution 10

Let $QR=x$ and $PR=y$

Then $x+y=25$

$y=25-x$

Now by Pythagoras theorem

$$x^2 + 25 = y^2$$

$$x^2 + 25 = (25-x)^2$$

Solving it ,we get

$$X=12 \text{ cm}$$

Then $y=25-12=13 \text{ cm}$

Now $\sin P = 12/13$

$\cos P = 5/13$

$\tan P = 12/5$

Solution 11

- i) False . The value of $\tan A$ increase from 0 to ∞ .
- ii) True . The value of $\sec A$ increase from 1 to ∞ .
- iii) False . $\cos A$ is the abbreviation used for the cosine of angle A
- iv) False . $\cot A$ is one symbol. We cannot separate it
- v) False. The value of $\sin \theta$ always lies between 0 and 1 and $4/3 > 1$