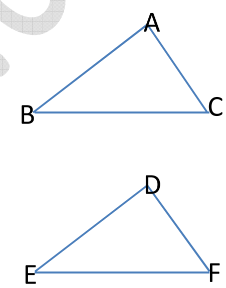
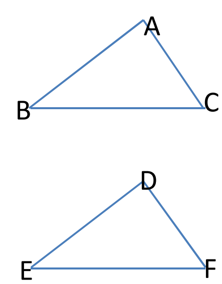
Quadrilaterals Exercise 1

The question is these will extensively Triangle congruence to prove various facts

I am giving here the short summary of what we learned in Triangles

N	Criterion	Description	Figures and expression
1	Side angle Side (SAS) congruence	• Two triangles are congruent if the two sides and included angles of one triangle is equal to the two sides and included angle • It is an axiom as it cannot be proved so it is an accepted truth • ASS and SSA type two triangles may not be congruent always	 **If following condition** **AB=DE, BC=EF** $\angle \boldsymbol{B} = \angle \boldsymbol{E}$ **Then** $ABC \cong DEF$
2	Angle side angle (ASA) congruence	• Two triangles are congruent if the two angles and included side of one triangle is equal to the corresponding angles and side • It is a theorem and can be proved	

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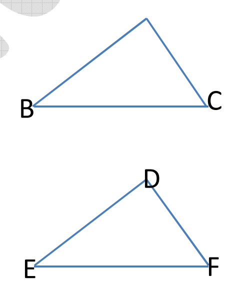
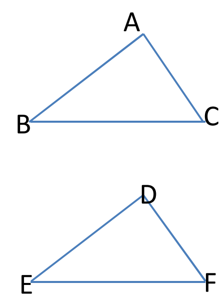
			If following condition **BC=EF** $\angle \boldsymbol{B} = \angle \boldsymbol{E}, \angle \boldsymbol{C} = \angle \boldsymbol{F}$ **Then** $ABC \cong DEF$
3	Angle angle side(AAS) congruence	• Two triangles are congruent if the any two pair of angles and any side of one triangle is equal to the corresponding angles and side • It is a theorem and can be proved	 **If following condition** **BC=EF** $\angle \boldsymbol{A} = \angle \boldsymbol{D}, \angle \boldsymbol{C} = \angle \boldsymbol{F}$ **Then** $ABC \cong DEF$
4	Side-Side-Side (SSS) congruence	• Two triangles are congruent if the three sides of one triangle is equal to the three sides of the another	

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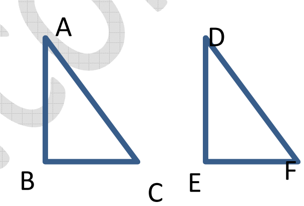
			If following condition **BC=EF,AB=DE,DF=AC** **Then** $ABC \cong DEF$
5	Right angle – hypotenuse-side(RHS) congruence	• Two right triangles are congruent if the hypotenuse and a side of the one triangle are equal to corresponding hypotenuse and side of the another	 **If following condition** **AC=DF,BC=EF** **Then** $ABC \cong DEF$

Question 1:
The angles of quadrilateral are in the ratio 3 : 5 : 9 : 13. Find all the angles of the Quadrilateral

Solution:

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Let the common ratio between the angles be y. Therefore, the angles will be 3y, 5y, 9y, and 13y respectively.

As the sum of all interior angles of a quadrilateral is 360^0

$3y + 5y + 9y + 13y = 360^0$

$30y = 360^0$

$y = 12^0$

Hence, the angles are

$3y = 3 \times 12 = 36^0$

$5y = 5 \times 12 = 60^0$

$9y = 9 \times 12 = 108^0$

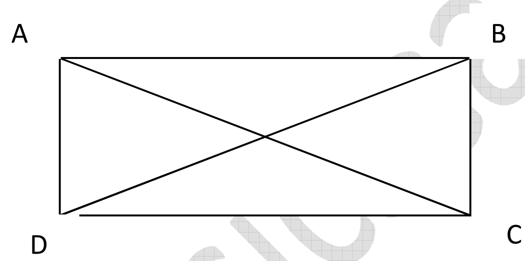
$13y = 13 \times 12 = 156^0$

Question 2

If the diagonals of a parallelogram are equal, then show that it is a rectangle

Solution:

Let ABCD is a parallelogram



Given

AC=BD

AD=BC and CD=AB (in parallelogram opposite side are equal)

To Prove:

Show that it is a rectangle

Proof

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In Δ ADC and ΔBCD

AD=BC

CD =CD

AC=BD

So by SSS congruence

$\triangle$ ADC$\cong$ $\triangle$ BCD

So $\angle$ ADC=$\angle$ BCD by CPCT (corresponding parts of the two congruent triangles)

But, we also have $\angle$ ADC+$\angle$ BCD=180^o (Co-interior angles because BC$\parallel$ AD)

So 2$\angle$ ADC=180

=> $\angle$ ADC=90

So $\angle$ BCD=90

So all the angles A, B, C, D are 90^0 .Hence rectangle

Question 3:

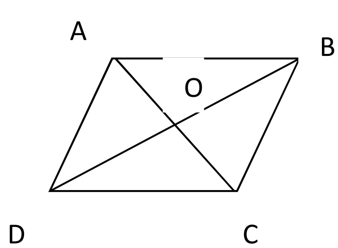
Show that if the diagonals of a quadrilateral bisect each other at right angles, then it is a rhombus.

Solution:

Given

Let ABCD be the quadrilateral and AC and BD are diagonal which bisect at right angles

OA = OC, OB = OD and $\angle$ AOB = $\angle$ BOC = $\angle$ COD = $\angle$ AOD = 90º



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To Prove : ABCD is rhombus

Proof:

Now, in ΔAOD and Δ COD
OA = OC (Diagonal bisects each other)
∠ AOD = ∠ COD (given)
OD = OD (common)
ΔAOD ≅ ΔCOD (by SAS congruence rule)
∴ AD = CD by CPCT (1)

Similarly we can prove that
AD = AB and CD = BC (2)
From equations (1) and (2), we can say that
AB = BC = CD = AD
Since all sides are equal, it is a rhombus

Question 4:
Show that the diagonals of a square are equal and bisect each other at right angles.

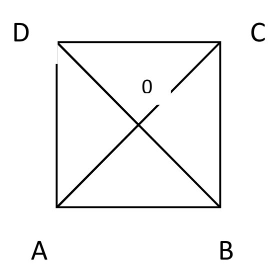
Solution:

Let ABCD is a square whose diagonal BD and AC intersect art O

Given

AB=BC=CD=AD

$\angle A = \angle B = \angle C = \angle D = 90$



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To prove: Diagonal are equal and bisect each other at right angle

OA=OC

OD=OB

AC=BD

$AC \perp BD$

Proof:

In △ ABC and △ DCB,
AB = DC (From Sides of a square are equal to each other)
∠ ABC =∠ DCB (All interior angles are of 90)
BC = CB (Common side)
△ ABC ≅ △ DCB (By SAS congruency)
AC = DB (By CPCT)
Hence, the diagonals of a square are equal in length.
In △ AOB and △ COD,
∠ AOB = ∠ COD (Vertically opposite angles)
∠ ABO = ∠ CDO (Alternate interior angles)
AB = CD (Sides of a square are always equal)
∴ △ AOB ≅ △ COD (By AAS congruence rule)
∴ AO = CO and OB = OD (By CPCT)
Hence, the diagonals of a square bisect each other.
In △ AOB and △ COB,
As we had proved that diagonals bisect each other, therefore,
AO = CO
AB = CB (Sides of a square are equal)
BO = BO (Common)
∴ △ AOB ≅ △ COB (By SSS congruency)
∴ ∠ AOB = ∠ COB (By CPCT)
However, ∠ AOB + ∠ COB = 180º (Linear pair)
2∠ AOB = 180º
∠ AOB = 90º
Hence, the diagonals of a square bisect each other at right angles.

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Question 5:

Show that if the diagonals of a quadrilateral are equal and bisect each other at right angles, then it is a square

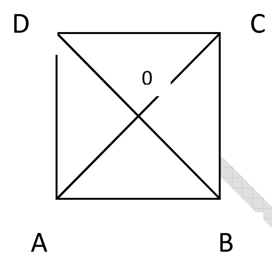
Solution:

Let ABCD is a quadrilateral whose diagonal BD and AC bisect each other at right angle at O

Given

AO=OC, BO=OD, AC=BD

∠ AOB=∠ COD



To Prove: ABCD is a square

Proof:

In △ AOB and △ COD
AO=CO (Given)
∠ AOB=∠ COD (Each given equal to 90°)
BO=DO (Given)
Therefore, by SAS congruence rule, △ AOB≅ △ COD.
⟹ ∠ OBA=∠ ODC (by CPCT)
But, these are alternate interior angles which means that AB∥ CD. (1)
Similarly, we can prove that BC∥ AD (2)
From (1) and (2), we can say that quadrilateral ABCD is a parallelogram. Hence, we have AB=CD and BC=AD because opposite sides of a parallelogram are equal. (3)
Now, in △ AOB and △ AOD
AO=AO (Common)
∠ AOB=∠ AOD (Each given equal to 90°)
OB=OD (Given)

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Therefore, by SAS congruence rule, we have △ AOB≅ △ AOD
⟹ AB=AD (by CPCT) (4)

In △ ACD and △ BDC
AC=BD (Given)
AD=BC (Proved above in (1))
CD=DC (Common)
Therefore, by SSS congruence rule, △ ACD≅ △ BDC

⟹ ∠ ADC=∠ BCD (Corresponding parts of congruent triangles are equal) (5)
But, we also have ∠ ADC+∠ BCD=180° (Co-Interior angles) (6)

From (5) and (6), we can say that
∠ ADC+∠ ADC=180°
⟹ 2∠ ADC=180°
⟹ ∠ ADC=180/2=90° (7)

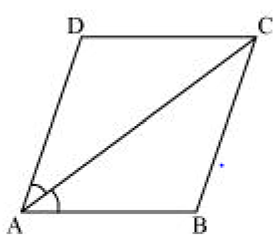
From (3), (4) and (7), we can say that ABCD is a parallelogram having all the sides equal and we have showed that it's one angle is equal to 90° which is enough to consider it a square. Therefore, ABCD is a square.

Question 6:

Diagonal AC of a parallelogram ABCD bisects ∠ A (see the given figure). Show that

(i) It bisects ∠ C also,

(ii) ABCD is a rhombus.



Solution:

i)

Given

ABCD is a parallelogram and AC bisect angle A

∠ DAC = ∠ BAC

To Prove: AC bisects ∠ C.

Proof:

As ABCD is a parallelogram

∠ DAC = ∠ BCA (Alternate interior angles) ... (1)

And ∠ BAC = ∠ DCA (Alternate interior angles) ... (2)

But it is given that AC bisects ∠ A.

∴ ∠ DAC = ∠ BAC ... (3)

From equations (1), (2) and (3), we have

∠ DAC = ∠ BCA =∠ BAC = ∠ DCA ... (4)

=> ∠ DCA = ∠ BCA

Hence, AC bisects ∠ C.

(ii)

To Prove: ABCD is a rhombus

Proof :

From equation (4), we have

∠ DAC = ∠ DCA

∴ DA = DC (side opposite to equal angles are equal)

But DA = BC and AB = CD (opposite sides of parallelogram)

∴ AB = BC = CD = DA

Hence, ABCD is rhombus

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Question 7:

ABCD is a rhombus. Show that diagonal AC bisects ∠ A as well as ∠ C and diagonal BD bisects ∠ B as well as ∠ D.

Solution:

Given: ABCD is a rhombus

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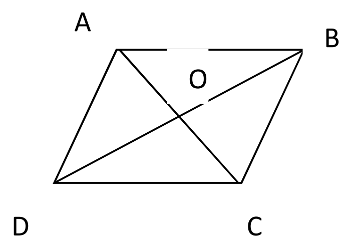

To Prove: Diagonal BD bisect angle B and D

Diagonal AC bisect angle A and C

Proof:

Let us join AC.
In △ ABC,
BC = AB (Sides of a rhombus are equal to each other)
∴ ∠ BAC = ∠ BCA (Angles opposite to equal sides of a triangle are equal)
However, ∠ BAC = ∠ DCA (Alternate interior angles for parallel lines AB and CD)
⟹ ∠ BCA= ∠ DCA
Therefore, AC bisects ∠ C.
Also, ∠ BCA= ∠ DAC (Alternate interior angles for || lines BC and DA)
⟹ ∠ BAC = ∠ DAC
Therefore, AC bisects ∠ A.
Similarly, it can be proved that BD bisects ∠ B and ∠ D as well.

Question 8:

ABCD is a rectangle in which diagonal AC bisects ∠ A as well as ∠ C. Show that:

(i) ABCD is a square
(ii) Diagonal BD bisects ∠ B as well as ∠ D.

Solution

Given

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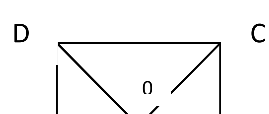

ABCD is a rectangle and $\angle DAC=\angle BAC$ and $\angle DCA=\angle BCA$

(i) **To Prove**: ABCD is a square

Proof:

In $\triangle$ ADC and $\triangle$ ABC
$\angle$ CAD=$\angle$ CAB (Given)
AC=AC (Common)
$\angle$ DCA=$\angle$ BCA (Given)
Therefore, by ASA congruence rule, $\triangle ADC \cong \triangle ABC$
$\Longrightarrow$ AD=AB (by CPCT) (2)

From (1) and (2), we can say that ABCD is a rectangle having all the sides equal. It means that ABCD is a square.

i) To prove diagonal BD bisects $\angle$ B as well as $\angle$ D.

In solution (i), we have showed that ABCD is a square.
Now in $\triangle$ CBD and $\triangle$ ABD
BC=BA (Sides of square are equal)
BD=BD (Common)
CD=AD (Sides of square are equal)
Therefore, by SSS congruence rule, $\triangle CBD \cong \triangle ABD$
$\Longrightarrow \angle CBD=\angle ABD$ (by CPCT) (3)
And, $\angle CDB=\angle ADB$ (by CPCT) (4)
From (3) and (4), we can say that BD bisects $\angle$ B as well as $\angle$ D.

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