

# Inequalities Exercise 1

---

**Question 1:**

Solve  $24x < 100$ , when (i)  $x$  is a natural number (ii)  $x$  is an integer

**Question 2:**

Solve  $-12x > 30$ , when

(i)  $x$  is a natural number (ii)  $x$  is an integer

**Question 3:**

Solve  $5x - 3 < 7$ , when

(i)  $x$  is an integer (ii)  $x$  is a real number

**Question 4:**

Solve  $3x + 8 > 2$ , when

(i)  $x$  is an integer (ii)  $x$  is a real number

**Question 5:**

Solve the given inequality for real  $x$ :  $4x + 3 < 5x + 7$

**Question 6:**

Solve the given inequality for real  $x$ :  $3(x - 1) \leq 2(x - 3)$

**Question 7:**

Solve the given inequality for real  $x$ :  $3x - 7 > 5x - 1$

**Question 8**

Solve the given inequality for real  $x$ :  $3(2 - x) \geq 2(1 - x)$

**Question 9**

Solve the given inequality for real  $x$ :

$$x + \frac{x}{2} + \frac{x}{3} < 11$$

**Question 10**

Solve the given inequality for real x:

$$\frac{x}{3} > \frac{x}{2} + 1$$

**Question 11**

Solve the given inequality for real x:

$$\frac{3(x-2)}{5} \leq \frac{5(2-x)}{3}$$

**Question 12:**

Solve the given inequality for real x:

$$\frac{1}{2} \left( \frac{3x}{4} + 4 \right) \geq \frac{1}{3} (x - 6)$$

**Question 13:**

Solve the given inequality for real x:  $2(2x + 3) - 10 < 6(x - 2)$

**Question 14**

Solve the given inequality for real x:  $37 - (3x + 5) \geq 9x - 8(x - 3)$

**Question 15**

Solve the given inequality for real x:

$$\frac{x}{4} < \frac{(5x - 2)}{3} - \frac{(7x - 3)}{5}$$

**Question 16**

Solve the given inequality for real x:

$$\frac{(2x - 1)}{3} < \frac{(3x - 2)}{4} - \frac{(2 - x)}{5}$$

**Question 17**

Solve the given inequality and show the graph of the solution on number line:  $3x - 2 < 2x + 1$

**Question 18**

Solve the given inequality and show the graph of the solution on number line:  $5x - 3 \geq 3x - 5$

**Question 19**

Solve the given inequality and show the graph of the solution on number line:  $3(1 - x) < 2(x + 4)$

**Question 20**

Solve the given inequality and show the graph of the solution on number line:

$$\frac{x}{2} > \frac{(5x - 2)}{3} - \frac{(7x - 3)}{5}$$

**Question 21:**

Ravi obtained 70 and 75 marks in first two unit test. Find the minimum marks he should get in the third test to have an average of at least 60 marks.

**Question 22:**

To receive Grade 'A' in a course, one must obtain an average of 90 marks or more in five examinations (each of 100 marks). If Sunita's marks in first four examinations are 87, 92, 94 and 95, find minimum marks that Sunita must obtain in fifth examination to get grade 'A' in the course.

**Question 23:**

Find all pairs of consecutive odd positive integers both of which are smaller than 10 such that their sum is more than 11

**Question 24**

Find all pairs of consecutive even positive integers, both of which are larger than 5 such that their sum is less than 23

**Question 25:**

The longest side of a triangle is 3 times the shortest side and the third side is 2 cm shorter than the longest side. If the perimeter of the triangle is at least 61 cm, find the minimum length of the shortest side

**Question 26**

A man wants to cut three lengths from a single piece of board of length 91 cm. The second length is to be 3 cm longer than the shortest and the third length is to be twice as long as the shortest. What are the possible lengths of the shortest board if the third piece is to be at least 5 cm longer than the second? [Hint: If  $x$  is the length of the shortest board, then  $x$ ,  $(x + 3)$  and  $2x$  are the lengths of the second and third piece, respectively. Thus,  $x = (x + 3) + 2x \leq 91$  and  $2x \geq (x + 3) + 5$ ]

**Solution 1:**

The given inequality is  $24x < 100$ .

Dividing both the side by the positive number 24

$$x < 100/24$$

$$x < 25/6$$

$$x < 4.16$$

(i) It is evident that 1, 2, 3, and 4 are the only natural numbers less than

Hence, in this case, the solution set is  $\{1, 2, 3, 4\}$ .

(ii) The integers less than are  $\dots-3, -2, -1, 0, 1, 2, 3, 4$ .

Solution set =  $\{\dots-3, -2, -1, 0, 1, 2, 3, 4\}$ .

**Solution 2:**

The given inequality is  $-12x > 30$

Dividing both the sides by -12

Also as we are dividing by negative integer, the inequality sign will reverse

$$x < -30/12$$

$$x < -5/2$$

(i) There is no natural number less than  $-5/2$

Thus there is no solution of the given inequality.

(ii) The integers less than are ...,  $-5, -4, -3$ .

Solution set =  $\{\dots, -5, -4, -3\}$ .

### **Solution 3:**

The given inequality is

$$5x - 3 < 7$$

$$5x - 3 + 3 < 7 + 3 \text{ (adding 3 on both the sides)}$$

$$5x < 10$$

$$x < 2 \text{ (Dividing both the sides by 5)}$$

(i) The integers less than 2 are ...,  $-4, -3, -2, -1, 0, 1$ .

Hence, in this case, the solution set is  $\{\dots, -4, -3, -2, -1, 0, 1\}$ .

(ii) When  $x$  is a real number, the solutions of the given inequality are given by  $x < 2$ ,

Solution set =  $(-\infty, 2)$ .

### **Solution 4:**

The given inequality is

$$3x + 8 > 2$$

$$3x + 8 - 8 > 2 - 8$$

$$3x > -6$$

$$x > -2$$

(i) The integers greater than  $-2$  are  $-1, 0, 1, 2, \dots$

Hence, in this case, the solution set is  $\{-1, 0, 1, 2, \dots\}$ .

(ii) When  $x$  is a real number, the solutions of the given inequality are all the real

Numbers, which are greater than  $-2$ .

Solution set =  $(-2, \infty)$ .

**Solution 5:**

$$4x + 3 < 5x + 7$$

$$4x + 3 - 7 < 5x + 7 - 7$$

$$4x - 4 < 5x$$

$$4x - 4 - 4x < 5x - 4x$$

$$-4 < x$$

Solution set =  $(-4, \infty)$ .

**Solution 6:**

$$3x - 7 > 5x - 1$$

$$3x - 7 + 7 > 5x - 1 + 7 \text{ (Adding 7 on both sides)}$$

$$3x > 5x + 6$$

$$3x - 5x > 5x + 6 - 5x \text{ (Subtracting 5x on both sides)}$$

$$-2x > 6 \text{ (Dividing -2 on both the sides and reversing the inequality sign)}$$

$$x < -3$$

Solution set =  $(-\infty, -3)$ .

**Solution 7:**

$$3(x - 1) \leq 2(x - 3)$$

$$3x - 3 \leq 2x - 6$$

$$3x - 3 + 3 \leq 2x - 6 + 3 \text{ (Adding 3 on both sides)}$$

$$3x \leq 2x - 3$$

$$3x - 2x \leq 2x - 3 - 2x \text{ (Subtracting 2x on both sides)}$$

This material is created by <http://physicscatalyst.com/> and is for your personal and non-commercial use only.

$$x \leq -3$$

Solution set  $=(-\infty, -3]$ .

**Solution 8:**

$$3(2 - x) \geq 2(1 - x)$$

$$6 - 3x \geq 2 - 2x$$

$$6 - 3x + 2x \geq 2 - 2x + 2x \text{ (Adding } 2x \text{ on both sides)}$$

$$6 - x \geq 2$$

$$6 - x - 6 \geq 2 - 6 \text{ (Subtracting } 6 \text{ on both sides)}$$

$$-x \geq -4$$

$$x \leq 4 \text{ (Dividing } -1 \text{ on both the sides and reversing the inequality sign)}$$

Solution set  $=(-\infty, 4]$ .

**Solution 9:**

$$x + \frac{x}{2} + \frac{x}{3} < 11$$

$$x\left(1 + \frac{1}{2} + \frac{1}{3}\right) < 11$$

$$x\left(\frac{11}{6}\right) < 11$$

$$\frac{11x}{6} < 11$$

$$x < 6$$

Solution set  $=(-\infty, 6)$ .

**Solution 10**

$$\frac{x}{3} > \frac{x}{2} + 1$$

$$\frac{x}{3} - \frac{x}{2} > 1$$

$$-\frac{x}{6} > 1$$

$$x < -6$$

Solution set  $=(-\infty, -6)$ .

**Solution 11:**

$$\frac{3(x-2)}{5} \leq \frac{5(2-x)}{3}$$

$$\frac{3x}{5} - \frac{6}{5} \leq \frac{10}{3} - \frac{5x}{3}$$

$$\frac{3x}{5} + \frac{5x}{3} \leq \frac{10}{3} + \frac{6}{5}$$

$$\frac{34x}{15} \leq \frac{68}{15}$$

$$x \leq 2$$

Solution set  $=(-\infty, 2]$ .

**Solution 12:**

$$\frac{1}{2} \left( \frac{3x}{5} + 4 \right) \geq \frac{1}{3} (x - 6)$$

$$3 \left( \frac{3x}{5} + 4 \right) \geq 2(x - 6)$$

$$\left( \frac{9x}{5} + 12 \right) \geq (2x - 12)$$

$$\frac{9x}{5} - 2x \geq -12 - 12$$

$$\frac{9x - 10x}{5} \geq -24$$

$$120 \geq x$$

Solution set  $=(-\infty, 120]$ .

**Solution 13:**

$$2(2x + 3) - 10 < 6(x - 2)$$

$$4x + 6 - 10 < 6x - 12$$

This material is created by <http://physicscatalyst.com/> and is for your personal and non-commercial use only.



$$-4+12 < 6x-4x$$

$$8 < 2x$$

$$4 < x$$

Solution set =  $[4, \infty)$ .

#### Solution 14

$$37 - (3x + 5) \geq 9x - 8(x - 3)$$

$$37 - 3x - 5 \geq 9x - 8x + 24$$

$$32 - 24 > x + 3x$$

$$8 > 4x$$

$$2 > x$$

Solution set =  $(-\infty, 2]$ .

#### Solution 15:

$$\frac{x}{4} < \frac{(5x-2)}{3} - \frac{(7x-3)}{5}$$

$$\frac{x}{4} < \frac{5(5x-2) - 3(7x-3)}{15}$$

$$\frac{x}{4} < \frac{25x-10-21x+9}{15}$$

$$\frac{x}{4} < \frac{4x-1}{15}$$

$$15x < 16x - 4$$

$$4 < x$$

Solution set =  $(4, \infty)$ .

#### Solution 16

$$\frac{(2x-1)}{3} \geq \frac{(3x-2)}{4} - \frac{(2-x)}{5}$$

$$\frac{(2x-1)}{3} \geq \frac{5(3x-2) - 4(2-x)}{20}$$

$$\frac{(2x-1)}{3} \geq \frac{19x-18}{20}$$

$$20(2x-1) \geq 3(19x-18)$$

$$40x-20 \geq 57x-54$$

$$34 \geq 17x$$

$$2 \geq x$$

Solution set  $=(-\infty, 2]$ .

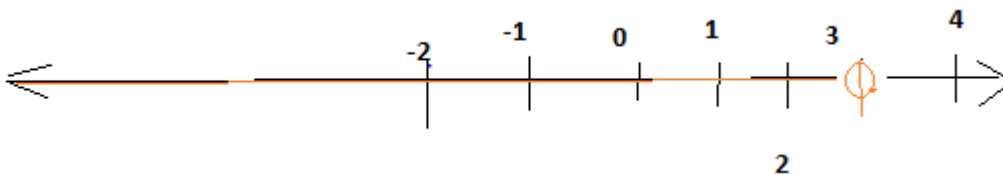
#### Solution 17

$$3x - 2 < 2x + 1$$

$$\Rightarrow 3x - 2x < 1 + 2$$

$$\Rightarrow x < 3$$

The graphical representation of the solutions of the given inequality is as follows.



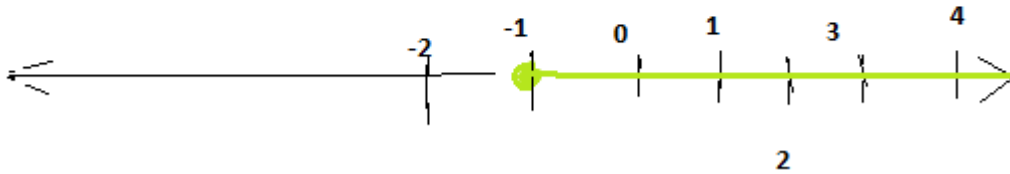
#### Solution 18:

$$5x - 3 \geq 3x - 5$$

$$2x \geq -2$$

$$x \geq -1$$

The graphical representation of the solutions of the given inequality is as follows.


**Solution 19**

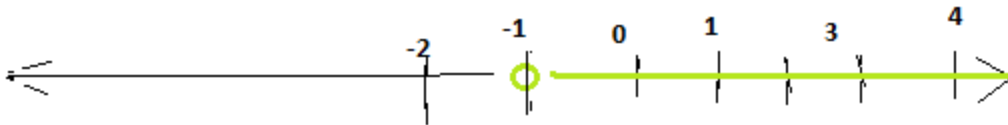
$$3(1-x) < 2(x+4)$$

$$3-3x < 2x+8$$

$$3-8 < 5x$$

$$-1 < x$$

The graphical representation of the solutions of the given inequality is as follows.


**Solution 20:**

$$\frac{x}{2} \geq \frac{5x-2}{3} - \frac{7x-3}{5}$$

$$\frac{x}{2} \geq \frac{5(5x-2)-3(7x-3)}{15}$$

$$\frac{x}{2} \geq \frac{4x-1}{15}$$

$$x \geq \frac{-2}{7}$$

**Solution 21:**

Let x be the marks obtained by Ravi in the third unit test.

Since the student should have an average of at least 60 marks,

$$\frac{70+75+x}{3} \geq 60$$

$$145 + x \geq 180$$

$$x \geq 35$$

Thus, the student must obtain a minimum of 35 marks to have an average of at least 60 marks.

**Solution 22:**

Let  $x$  be the marks obtained by Sunita in the fifth examination.

In order to receive grade 'A' in the course, she must obtain an average of 90 marks or more in five examinations.

Therefore,

$$\frac{87+92+94+95+x}{5} \geq 90$$

$$x \geq 82$$

Thus, Sunita must obtain greater than or equal to 82 marks in the fifth examination.

**Solution 23:**

Let  $y$  be the smaller of the two consecutive odd positive integers.

Then, the other integer is  $y + 2$ .

Since both the integers are smaller than 10,

$$y + 2 < 10$$

$$\Rightarrow y < 10 - 2$$

$$\Rightarrow y < 8 \dots (i)$$

Also, the sum of the two integers is more than 11.

$$\therefore y + (y + 2) > 11$$

$$2y + 2 > 11$$

$$2y > 11 - 2$$

$$2y > 9$$

From (i) and (ii), we obtain .

Since  $y$  is an odd number,  $y$  can take the values, 5 and 7.

Thus, the required possible pairs are (5, 7) and (7, 9).

**Solution 24:**

Let  $y$  be the smaller of the two consecutive even positive integers. Then, the other integer is  $y + 2$ .

Since both the integers are larger than 5,

$$y > 5$$

Also, the sum of the two integers is less than 23.

$$y + (y + 2) < 23$$

$$2y + 2 < 23$$

$$2y < 23 - 2$$

$$2y < 21$$

$$y < 10.5$$

But we know that  $y > 5$

So

$$5 < y < 10.5.$$

Since  $y$  is an even number,  $y$  can take the values, 6, 8, and 10.

Thus, the required possible pairs are (6, 8), (8, 10), and (10, 12).

**Solution 25:**

Let the length of the shortest side of the triangle be  $z$  cm.

Then, length of the longest side =  $3z$  cm

Length of the third side =  $(3z - 2)$  cm

This material is created by <http://physicscatalyst.com/> and is for your personal and non-commercial use only.

Since the perimeter of the triangle is at least 61 cm,

$$z + 3z + (3z - 2) \geq 61$$

$$z \geq 9$$

Thus, the minimum length of the shortest side is 9 cm.

**Solution 26:**

Let the length of the shortest piece be  $z$  cm. Then, length of the second piece and the third piece are  $(z + 3)$  cm and  $2z$  cm respectively.

Since the three lengths are to be cut from a single piece of board of length 91 cm,

$$z + (z + 3) + 2z \leq 91$$

$$4z + 3 \leq 91$$

$$4z \leq 91 - 3$$

$$4z \leq 88$$

$$z \leq 22$$

Also, the third piece is at least 5 cm longer than the second piece.

$$\therefore 2z \geq (z + 3) + 5$$

$$2z \geq z + 8$$

$$z \geq 8$$

From both the results

$$8 \leq z \leq 22$$

Thus, the possible length of the shortest board is greater than or equal to 8 cm but less than or equal to 22 cm.