

NCERT SOLUTIONS OF Factorization

Exercise 2

Question 1

Factorize the following expressions.

(i) $a^2 + 8a + 16$

(ii) $p^2 - 10p + 25$

(iii) $25m^2 + 30m + 9$

(iv) $49y^2 + 84yz + 36z^2$

(v) $4x^2 - 8x + 4$

(vi) $121b^2 - 88bc + 16c^2$

(vii) $(l + m)^2 - 4lm$

(viii) $a^4 + 2a^2b^2 + b^4$

Answer

We have to make use of following identities to factorize them

$$(a+b)^2 = a^2 + b^2 + 2ab$$

$$(a-b)^2 = a^2 + b^2 - 2ab$$

$$a^2 - b^2 = (a-b)(a+b)$$

$$\begin{aligned} \text{i)} \quad & a^2 + 8a + 16 \\ & = a^2 + 2 \times a \times 4 + 4^2 \end{aligned}$$

So from first identity, it can be written as

$$=(a+4)^2$$

$$\begin{aligned} \text{ii)} \quad & p^2 - 10p + 25 \\ & = p^2 - 2 \times p \times 5 + 5^2 \end{aligned}$$

So from second identity, it can be written as

$$=(p-5)^2$$

$$\begin{aligned} \text{iii)} \quad & 25m^2 + 30m + 9 \\ & = (5m)^2 + 2 \times 5m \times 3 + 3^2 \end{aligned}$$

So from first identity, it can be written as

$$=(5m+3)^2$$

$$\begin{aligned} \text{iv)} \quad & 49y^2 + 84yz + 36z^2 \\ & = (7y)^2 + 2 \times 7y \times 6z + (6z)^2 \end{aligned}$$

So from first identity, it can be written as

$$=(7y+6z)^2$$

$$\begin{aligned} \text{v)} \quad & 4x^2 - 8x + 4 \\ & = (2x)^2 - 2 \times 2x \times 2 + 2^2 \end{aligned}$$

So from second identity, it can be written as

$$= (2x-2)^2$$

$$= 4(x-1)^2 \text{ taking common factor 2 out of square}$$

$$\text{vi) } 121b^2 - 88bc + 16c^2$$

$$= (11b)^2 - 2 \times 11b \times 4c + (4c)^2$$

So from second identity, it can be written as

$$= (11b-4c)^2$$

$$\text{vii) } (l + m)^2 - 4lm$$

$$= l^2 + m^2 + 2lm - 4lm$$

$$= l^2 + m^2 - 2lm$$

So from second identity, it can be written as

$$= (l-m)^2$$

$$\text{viii) } a^4 + 2a^2b^2 + b^4$$

$$= (a^2)^2 + 2a^2b^2 + (b^2)^2$$

So from first identity, it can be written as

$$= (a^2+b^2)^2$$

Question 2

Factorize.

$$\text{(i) } 4p^2 - 9q^2$$

$$\text{(ii) } 63a^2 - 112b^2$$

$$\text{(iii) } 49x^2 - 36$$

$$\text{(iv) } 16x^5 - 144x^3$$

$$\text{(v) } (l + m)^2 - (l - m)^2$$

$$\text{(vi) } 9x^2 y^2 - 16$$

$$\text{(vii) } (x^2 - 2xy + y^2) - z^2$$

(viii) $25a^2 - 4b^2 + 28bc - 49c^2$

Answer

We have to make use of following identities to factorize them

$$(a+b)^2 = a^2 + b^2 + 2ab$$

$$(a-b)^2 = a^2 + b^2 - 2ab$$

$$a^2 - b^2 = (a-b)(a+b)$$

i)
$$\begin{aligned} 4p^2 - 9q^2 \\ = (2p)^2 - (3q)^2 \end{aligned}$$

So from third identity, it can be written as

$$= (2p-3q)(2p+3q)$$

ii)
$$\begin{aligned} 63a^2 - 112b^2 \\ = 7(9a^2 - 16b^2) \\ = 7[(3a)^2 - (4b)^2] \end{aligned}$$

So from third identity, it can be written as

$$= 7(3a-4b)(3a+4b)$$

iii)
$$\begin{aligned} 49x^2 - 36 \\ = (7x)^2 - (6)^2 \end{aligned}$$

So from third identity, it can be written as

$$= (7x-6)(7x+6)$$

$$\text{iv) } 16x^5 - 144x^3$$

$$= x^3(16x^2 - 144)$$

$$= x^3(4x+12)(4x-12)$$

$$\begin{aligned} \text{v) } & (l+m)^2 - (l-m)^2 \\ &= [l+m+l-m][l+m-l+m] \\ &= 2l \times 2m \\ &= 4lm \end{aligned}$$

$$\begin{aligned} \text{vi) } & 9x^2y^2 - 16 \\ &= (3xy-4)(3xy+4) \end{aligned}$$

$$\text{vii) } (x^2 - 2xy + y^2) - z^2$$

$$= (x-y)^2 - z^2 \quad \text{as } (a-b)^2 = a^2 + b^2 - 2ab$$

$$\text{Now as } a^2 - b^2 = (a-b)(a+b)$$

$$= (x-y+z)(x-y-z)$$

$$\begin{aligned} \text{viii) } & 25a^2 - 4b^2 + 28bc - 49c^2 \\ & \text{Factorizing each term} \\ &= (5a)^2 - (2b)^2 + 2 \times 2b \times 7c - (7c)^2 \\ & \text{Rearranging the terms} \\ &= (5a)^2 - [(2b)^2 - 2 \times 2b \times 7c + (7c)^2] \end{aligned}$$

$$\text{Now as } (a-b)^2 = a^2 + b^2 - 2ab$$

$$= (5a)^2 - (2b-7c)^2$$

$$= (5a-2b+7c)(5a+2b-7c)$$

Question 3

Factorize the expressions.

(i) $ax^2 + bx$

(ii) $7p^2 + 21q^2$

(iii) $2x^3 + 2xy^2 + 2xz^2$

(iv) $am^2 + bm^2 + bn^2 + an^2$

(v) $(lm + l) + m + 1$

(vi) $y(y + z) + 9(y + z)$

(vii) $5y^2 - 20y - 8z + 2yz$

(viii) $10ab + 4a + 5b + 2$

(ix) $6xy - 4y + 6 - 9x$

Answer

i) $ax^2 + bx$
 $=x(ax+b)$

ii) $7p^2 + 21q^2$
 $=7(p^2+3q^2)$

iii) $2x^3 + 2xy^2 + 2xz^2$
 $=2x(x^2+y^2+z^2)$

iv) $am^2 + bm^2 + bn^2 + an^2$
Rearranging the terms
 $= am^2+ an^2+ bm^2 + bn^2$
 $=a(m^2 +n^2) +b(m^2 +n^2)$
 $=(a+b) (m^2 +n^2)$

v) $(lm + l) + m + 1$
 $=l(m+1) +1(m+1)$
 $=(l+1)(m+1)$

vi) $y(y + z) + 9(y + z)$
 $= (y+z)(y+9)$

vii) $5y^2 - 20y - 8z + 2yz$

$$=5y(y-4) + 2z(y-4)$$

$$=(5y+2z)(y-4)$$

$$\text{viii) } 10ab + 4a + 5b + 2$$

$$=2a(5b+2) + 1(5b+2)$$

$$=(2a+1)(5b+2)$$

$$\text{ix) } 6xy - 4y + 6 - 9x$$

$$=2y(3x-2) + 3(2-3x)$$

$$=2y(3x-2) - 3(3x-2)$$

$$=(2y-3)(3x-2)$$

Question 4

Factorize.

(i) $a^4 - b^4$

(ii) $p^4 - 81$

(iii) $x^4 - (y + z)^4$

(iv) $x^4 - (x - z)^4$

(v) $a^4 - 2a^2b^2 + b^4$

Answer:

i) $a^4 - b^4 = (a^2 + b^2)(a^2 - b^2)$

ii) $p^4 - 81$

$$=(p^2+9)(p^2-9)$$

iii) $x^4 - (y + z)^4$

$$= (x^2 + (y+z)^2)(x^2 - (y+z)^2)$$

$$= (x^2 + (y+z)^2)[(x+y+z)(x-y-z)]$$

iv) $x^4 - (x-z)^4$

$$= (x^2 - (x-z)^2)(x^2 + (x-z)^2)$$

$$= [(x+x-z)(x-x+z)][(x^2 + (x-z)^2)]$$

$$= z(2x-z)[(x^2 + (x-z)^2)]$$

v) $a^4 - 2a^2b^2 + b^4$

$$= (a^2 - b^2)^2$$

Question 5

Factorize the following expressions.

(i) $p^2 + 6p + 8$

(ii) $q^2 - 10q + 21$

(iii) $p^2 + 6p - 16$

Answer

i) $p^2 + 6p + 8$

$$= p(p+6) + 8$$

ii) $q^2 - 10q + 21$

$$= q(q-10) + 21$$

iii) $p^2 + 6p - 16$

$$= p(p+6) - 16$$