How to solve the Center of mass Problems

What is center of Mass & How to Solve the Center of Mass Problems

1) *Center of Mass is a important concept in a system of many particles.* ***Centre of mass is the point where whole mass of the system can be supposed to be concentrated and motion of the system can be defined in terms of the centre of mass*** . It is the mass weighted average of its components

2) *It can be calculated as*

$$X_{cm} = \frac{m_1 x_1 + m_2 x_2 + m_3 x_3 + \dots + m_n x_n}{m_1 + m_2 + m_3 + \dots + m_n} = \frac{1}{m}\sum_{i=1}^{n} m_i x_i \qquad (11)$$

$$Y_{cm} = \frac{m_1 y_1 + m_2 y_2 + m_3 y_3 + \dots + m_n y_n}{m_1 + m_2 + m_3 + \dots + m_n} = \frac{1}{m}\sum_{i=1}^{n} m_i y_i \qquad (12)$$

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3) *The co-ordinates of Center of mass depends on refrence frame.But it physical location is independent of choice of the frame*

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- *Velocity and acceleration of the center of mass is given by*

$$V_{cm} = \sum_{1}^{n} \frac{m_i v_i}{m_i} \qquad a_{cm} = \sum_{1}^{n} \frac{m_i a_i}{m_i}$$

- *In the absence of external force velocity of centre of mass of the system remains constant or we can say that centre of mass moves with the constant velocity in absence of external force. ie no acceleration*

- The motion of the system of particles can be understood easily as translational motion of the center of mass and rotation of particles around center of mass

Four particles of same mass lies in x-y plane.The (x,y) coordinates of their positions are (2,2) (3,3),(-1,2) and (-1,-1) respectively. Find the position of the center of mass of the system

Solution

1) It is a simple problem ,where we know the coordinates of the four particle and we need to find CM of the system.It can be simple calculated by the formula given below

$$X_{cm} = \frac{m_1x_1 + m_2x_2 + m_3x_3 + \dots\dots\dots + m_nx_n}{m_1 + m_2 + m_3 + \dots\dots\dots + m_n} = \frac{1}{m}\sum_{i=1}^{n} m_ix_i \qquad (11)$$

$$Y_{cm} = \frac{m_1y_1 + m_2y_2 + m_3y_3 + \dots\dots\dots + m_ny_n}{m_1 + m_2 + m_3 + \dots\dots\dots + m_n} = \frac{1}{m}\sum_{i=1}^{n} m_iy_i \qquad (12)$$

$$Z_{cm} = \frac{m_1z_1 + m_2z_2 + m_3z_3 + \dots\dots\dots + m_nz_n}{m_1 + m_2 + m_3 + \dots\dots\dots + m_n} = \frac{1}{m}\sum_{i=1}^{n} m_iz_i \qquad (13)$$

Solution continued

Here we are talking about x-y plane,so we can ignore z coordinates
Also Let us assume m be the mass of each particle
Now

$X=(2m+3m-m-m)/(m+m+m+m)=3/4$

$Y=(2m+3m+2m-m)/(m+m+m+m)=6/4=3/2$

So $(x,y) =(3/4,3/2)$

A rocket following a parabolic path in air suddenly explodes into two parts What can be told about the center of mass of the system after explosion

a) Vertically downwards
b) Horizontally
c) It will move along the same parabolic path which the intact shell was moving
d) It will move along the tangent to the parabolic path at the point of explosion

Solution

We know that center of mass motion depends on the net external force. Since the net external force(Gravity) remains constant before the explosion and after the explosion,the center of mass will keep on moving in the same direction.

Answer (c)

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