Differential equations

In each of the Exercises 1 to 10 verify that the given functions (explicit or implicit) is a solution of the corresponding differential equation

Question 1:

$y = e^x + 1$: $y'' - y' = 0$

Solution

$y = e^x + 1$

Differentiating both sides of this equation with respect to x, we get:

$y' = e^x$

Now, differentiating equation (1) with respect to x, we get:

$y'' = e^x$

Substituting the values of in the given differential equation, we get the L.H.S.

as:

$e^x - e^x = 0$

Thus, the given function is the solution of the corresponding differential equation.

Question 2:

$y = x^2 + 2x + C$: $(dy/dx) - 2x - 2 = 0$

Solution

Differentiating both sides of this equation with respect to x, we get:

$(dy/dx) = 2x + 2$

Substituting the value in the given differential equation, we get:

L.H.S. =

(dy/dx) -2x -2 =0

=(2x+2) -2x-2=0

= R.H.S.

So, the given function is the solution of the corresponding differential equation.

Question 3:

y= cos x + C : (dy/dx) +sin x =0

Solution

Differentiating both sides of this equation with respect to x, we get:

(dy/dx) = -sin x

Substituting the value in the given differential equation, we get:

L.H.S. =

(dy/dx) +sin x

=-sin x + sin x

=0

= R.H.S.

So, the given function is the solution of the corresponding differential equation.

Question 4:

$$y = \sqrt{1+x^2} \quad : \quad y' = \frac{xy}{1+x^2}$$

Solution

Differentiating both sides of the equation with respect to x, we get:

$$\frac{dy}{dx} = \frac{1}{2\sqrt{1+x^2}}\frac{d}{dx}(1+x^2)$$

$$\frac{dy}{dx} = \frac{x}{\sqrt{1+x^2}}$$

$\frac{dy}{dx} = \frac{x}{y}$

$\frac{dy}{dx} = \frac{x}{y} \times \frac{y}{y} = \frac{xy}{(1+x^2)}$

So, the given function is the solution of the corresponding differential equation.

Question 5:

$y = Ax : \quad x(dy/dx) = y \quad (x \neq 0)$

Solution

Differentiating both sides with respect to x, we get:

dy/dx = A

Substituting the value of in the given differential equation, we get:

LHS

=x(dy/dx)

=Ax

=y

=RHS

So, the given function is the solution of the corresponding differential equation.

Question 6:

$y = x \sin x \quad : \quad xy' = y + x\sqrt{x^2 - y^2} \ (x \neq 0 \text{ and } x > y \text{ or } x < -y)$

Solution

Differentiating both sides of this equation with respect to x, we get:

dy/dx = sin x + x cos x

Substituting the value in the given differential equation, we get:

LHS

$=x(dy/dx)$

$= x(\sin x + x \cos x)$

$=x \sin (x) + x^2 \cos (x)$ -(1)

Now we know that

$\cos^2 x = 1 – \sin^2 x$

$\cos (x) = \sqrt{1 – \sin^2 x}$

Now we know that

$y=x \sin x$ or $\sin (x) = y/x$

Therefore

$\cos (x) = \sqrt{[1 –(y/x)^2]}$ - (2)

Substituting the value from (2) in (1)

$=x \sin (x) + x^2 \sqrt{[1 –(y/x)^2]}$

$= y + x \sqrt{(x^2 – y^2)}$

=RHS

Hence, the given function is the solution of the corresponding differential equation.

Question 7:

$xy= \log y + c$: $dy/dx = y^2 /(1-xy)$, $xy \neq 0$

Solution

Differentiating both sides of this equation with respect to x, we get:

$y +x (dy/dx) = (1/y) (dy/dx)$

$y^2 + xy (dy/dx) = (dy/dx)$

or

$dy/dx = y^2 /(1-xy)$

Hence, the given function is the solution of the corresponding differential equation.

Question 8:

$y - \cos y = x : \quad (y \sin y + \cos y + x)\, y' = y$

Solution

Differentiating both sides of the equation with respect to x, we get:

(dy/dx) + sin y (dy/dx) =1

Or

dy/dx= 1/(1+sin y)

Substituting the value of in differential equation

LHS

$=(y \sin y + \cos y + x)\, y'$

$=(y\sin y + \cos y + y - \cos y)\, [1/(1+\sin y)]$

$= y(1+ \sin y)\, [1/(1+\sin y)]$

$= y$

$=RHS$

Hence, the given function is the solution of the corresponding differential equation.

Question 9:

x + y= $\tan^{-1}$ y : y^2 (dy/dx) + y^2 + 1 =0

Solution

Differentiating both sides of this equation with respect to x, we get:

1 + (dy/dx) = [1/(1+y^2)] (dy/dx)

1+ y^2 (dy/dx) + (dy/dx) + y^2 = (dy/dx)

Or

y^2 (dy/dx) + y^2 + 1 =0

So, the given function is the solution of the corresponding differential equation.

Question 10:

$y=\sqrt{(a^2 - x^2)}$ $x \in (-a,a)$: $x + y(dy/dx) = 0$ $(y \neq 0)$

Solution

Differentiating both sides of this equation with respect to x, we get:

$$\frac{dy}{dx} = \frac{1}{2\sqrt{a^2 - x^2}} \frac{d}{dx}(a^2 - x^2)$$

$$\frac{dy}{dx} = \frac{-2x}{2\sqrt{a^2 - x^2}}$$

$$\frac{dy}{dx} = \frac{-x}{\sqrt{a^2 - x^2}}$$

Now $y=\sqrt{(a^2 - x^2)}$

So

$$\frac{dy}{dx} = \frac{-x}{y}$$

$x + y(dy/dx) = 0$

So, the given function is the solution of the corresponding differential equation.

Question 11:

The numbers of arbitrary constants in the general solution of a differential equation of

fourth order are:

(A) 0

(B) 2

(C) 3

(D) 4

Solution

We know that the number of constants in the general solution of a differential equation

of order n is equal to its order.

Therefore, the number of constants in the general equation of fourth order differential equation is four.

Hence, the correct answer is D.

Question 12:

The numbers of arbitrary constants in the solution of a differential equation of third order are:

(A) 3

(B) 2

(C) 1

(D) 0

Solution

In a solution of a differential equation, there are no arbitrary constants.

Hence, the correct answer is D.