Probability

Question 1

Given below are events such as P(E) = 0.6, P(F) = 0.3 P(E∩F) = 0.2. Determine the value of P(E/F) and P(F|E)

Solution

Given that P(E) = 0.6, P(F) = 0.3, P (E∩F) = 0.2

P(E|F) = P(E∩F)/P(F) = 0.2/0.3 = 2/3

P(F|E) = P(F∩E)/ P(E) = 0.2/0.6 = 1/3

Question 2

Determine the value of P (A|B) if A & B are the events such that P(B) = 0.5 and P(A∩B) = 0.32

Solution

Given that P(B) = 0.5, P(A∩B) = 0.32.

P(A|B) = P(A∩B)/P(B)

= 0.32/0.5 **=** 16/25

Question 3

P(A) = 0.8, P(B) = 0.5, P(B|A) = 0.4. Determine the value of

(a) P(A∩B)

(b) P(A|B)

(c) P(A∪B)

Solution

Given

P(A) = 0.8, P(B) = 0.5, P(B|A) = 0.4

(a) P(A∩B)

= P(A)P(B|A)

Therefore, $P(A\cap B) = 0.32$

(b) $P(A|B) = P(A\cap B)/P(B) = 0.32/0.5$ **= 0.64**

(c) $P(A\cup B) = P(A)+P(B)–P(A\cap B)$

$= 0.8+0.5–32$ **= 0.98**

Question 4

Determine the value of P(AUB) if $2P(A) = P(B) = 5/13$ & $P(A|B) = 2/5$

Solution

Given

$2P(A) = P(B) = 5/13$ & $P(A|B) = 25$

$P(A) = 5/26$

$P(B) = 5/13$

$P(A|B) = 2/5$

$P(A\cap B)/P(B) = 2/5$

$P(A\cap B) = 2/5\times5/13$

$= 2/13$

We know that:

$P(A\cup B) = P(A)+P(B)–P(A\cap B)$

$= 5/26+5/13–2/13$

$= 11/26$

Question 5

If A and B are events such that $P(A) = 6/11$, $P(B) = 5/11$, $P(A\cup B) = 7/11$. Determine the value of

(a) $P(A\cap B)$

(b) $P(A|B)$

(c) $P(B|A)$

Solution

Given

$P(A) = 6/11$, $P(B) = 5/11$, $P(A\cup B) = 7/11$.

(a) $P(A\cup B) = P(A)+P(B)-P(A\cap B)$

$7/11 = 6/11 + 5/11 - P(A\cap B)$

$P(A\cap B) = 11/11 - 7/11$

$P(A\cap B) = 4/11$

(b) $P(A|B) = P(A\cap B)/ P(B)$

$= (4/11) / (5/11) = 4/5$

(c) $P(B|A) = P(A\cap B)/P(A)$

$= (4/11)/ (6/11) = 2/3$

Question 6

An experiment consists of tossing up of a coin three times. Determine the following:

(a) E: obtaining heads on third toss & F: obtaining heads from the first consecutive two tosses.

(b) E: obtaining at least two heads & F: obtaining at most two heads.

(c) E: obtaining at most two tails & F: obtaining at most one tail.

Determine P(E|F) in each of these cases

Solution

Given a coin is tossed thrice in an order to conduct an experiment.

Sample space (S) = {TTT, TTH, THT, THH, HTT, HTH, HHT, HHH}

(a) Number of favorable outcomes of event E = {TTH, THH, HTH, HHH}

Number of favorable outcomes of event F = {HHT, HHH}

E∩F = {HHH}

P(E∩F) =1/8

P(F) = 2/8=1/4

P(E|F) = P(E∩F)/P(F) = (1/8) / (1/4) = 1/2

(b) Number of favorable outcomes of event E = {THH, THH, HHT, HHH}

Number of favorable outcomes of event F = {TTT, TTH, THT, THH, HTT, HTH, HHT}

E∩F = {HHT, HTH, HHT}

P(E∩F) =3/8

P(F) = 7/8

P(E|F) = P(E∩F)/P(F) = (3/8) / (7/8) = 3/7

(c) Number of favorable outcomes of event E = {TTH, THT, THH, HTH, HTT, HHT, HHH}

Number of favorable outcomes of event F = {TTT, TTH, THT, THH, HTT, HHT, HTH}

E∩F = {TTH, THT, THH. HTH, HTT, HHT}

P(E∩F) =6/8

P(F) = 7/8

P(E|F) = P(E∩F)/P(F)= (6/8) / (7/8) = 6/7

Question 7

Two coins are tossed once, where

(a) E: obtaining tail on one coin & F: head appears in one coin

(b) E: tail does not appear & F: head does not appear

Solution

Two coins are tossed once

Sample space (S) = {TT, TH, HT, HH}

(a) Number of favorable outcomes of event E = {TH, HT}

Number of favorable outcomes of event F = {TH, HT}

E∩F = {TH, HT}

P(E∩F) = 2/4= 1/2

P(F) = 2/4=1/2

P(E|F) = P(E∩F)/ P(F)

= 1

(b) Number of favorable outcomes of event Q = {HH}

Number of favorable outcomes of event F = {TT}

E∩F = {ϕ}

P(E∩F) = 0

P(F) = 1/4

P(E|F) = P(E∩F)/ P(F)

= 0

Question 8

A die is thrown three times,

E: the number 4 appears during the third toss

F: 6 & 5 appears consecutively during the first two tosses

Solution

Given that a die is being tossed thrice.

Total number of elements in the sample space = 6x6x6=216

Favorable outcomes of event E =

{(6,1,4), (6,2,4), (6,3,4), (6,4,4), (6,5,4), (6,6,4)

(5,1,4), (5,2,4), (5,3,4), (5,4,4), (5,5,4), (5,6,4)

(4,1,4), (4,2,4), (4,3,4), (4,4,4), (4,5,4), (4,6,4)

(3,1,4), (3,2,4), (3,3,4), (3,4,4), (3,5,4), (3,6,4)

(2,1,4), (2,2,4), (2,3,4), (2,4,4), (2,5,4), (2,6,4)

(1,1,4), (1,2,4), (1,3,4), (1,4,4), (1,5,4), (1,6,4)}

Favorable outcomes of event F = {(6,5,6), (6,5,5), (6,5,4), (6,5,3), (6,5,2), (6,5,1)}

$E \cap F = \{(6,5,4)\}$

$P(F) = 6/216$

$P(E \cap F) = 1/216$

$P(E|F) = P(E \cap F)/P(F) = (1/216) / (6/216) = 1/6$

Question 9

Mother, father and son line up at random for a family picture

E: the son is placed in one of the end

F: the position of the father is in the middle

Solution

Mother, father and son line up at random for a family picture

Let us consider the father, mother and the child are denoted by F, M and S respectively.

Sample space = {SFM, SMF, FSM, FMS, MSF, MFS}

Favorable outcomes of event E = {SFM, SMF, FMS, MFS}

Favorable outcomes of event F = {SFM, MFS}

$P(F) = 2/6=1/3$

$P(E\cap F) = 2/6=1/3$

$P(E|F) = P(E\cap F)/P(F) = (1/3) / (1/3)$ **= 1**

Question 10

A black and a red dice are rolled.

(a) Determine the conditional probability of getting a sum of numbers greater than 9 such that the black die results in a 5

(b) Determine the conditional probability of getting a sum of numbers 8 such that red die results in numbers less than 4

Solution

Given A black and a red dice are rolled.

Total number of elements in the sample space **=** 6X6=36

(a) Favorable outcomes of event E = {(4,6), (5,5), (5,6), (6,4), (6,5), (6,6)}

Favorable outcomes of event F = {(5,1), (5,2), (5,3), (5,4), (5,5), (5,6)}

E∩F = {(5,6), (5,5)}

Conditional probability of getting a sum greater than 9, so that the black die results in 5.

P (E|F) = P(E∩F)/P(F)

= (2/36) / (6/36) = 1/3

b) Favorable outcomes of event E = {(2,6), (3,5), (4,4), (5,3), (6,2)}

Favorable outcomes of event F =

{(1,1), (1,2), (1,3), (2,1), (2,2), (2,3),

(3,1), (3,2), (3,3), (4,1), (4,2), (4,3)

(5,1), (5,2), (5,3), (6,1), (6,2), (6,3)}

E∩F = {(5,3), (6,2)}

Conditional probability of getting a sum of numbers 8 such that red die results in numbers less than 4.

P (E|F) = P(E∩F)/P(F)

= (2/36) / (18/36) = 1/9

Question 11

A fair die is rolled. Consider events E = {1,3,5}, F = {2,3} and G = {2,3,4,5}
Find
(i) P(E|F) and P (F|E)
(ii) P(E|G) and P(G|E)

(iii) P ((E ∪ F) |G) and P (E ∩ F) |G)

Solution

Given

E = {1,3,5}, F = {2,3} and G = {2,3,4,5}

Sample space = {6, 5, 4, 3, 2, 1}

Favorable outcomes of event E = {5, 3, 1}

Favorable outcomes of event F = {3, 2}

Favorable outcomes of event G = {5, 4, 3, 2}

Therefore, P(E) = 3/6=1/2

P(F) = 2/6=1/3

P(G) = 4/6=2/3

(a) $E \cap F$ = {3}

P ($E \cap F$) = 1/6

P(E|F) =P($E \cap F$)/P(F) = (1/6) / (1/3) =1/2
P(F|E) =P($E \cap F$)/ P(E) = (1/6) / (1/2) =1/3

(b) $E \cap G$ = {5, 3}

P ($E \cap G$) = 1/6

P(E|G) =P($E \cap G$)/ P(G)= (1/3) / (2/3) =1/2
P(G|E) =P($E \cap G$)/ P(E) = (1/3) / (1/)2=2/3

(c) $E \cup F$ = {5, 3, 2, 1}

($E \cup F$) $\cap$G = {5,3,2}

$E \cap F$ = {3}

($E \cap F$) $\cap$G= = {3}

P($E \cup F$) =4/6=2/3
P[($E \cup F$) $\cap$G] =3/6=1/2
P($E \cap F$) =1/6
P[($E \cup F$) |G] =P[($E \cup F$) $\cap$G]/P(G)= (1/2) / (2/3) =3/4

P[($E \cap F$) |G] =P[($E \cap F$) $\cap$G]/P(G)= (1/6) / (2/3) =1/4

Question 12

Assume that each of the children born in a family can either be a boy or a girl. If a family having 2 children, determine the conditional probability that both are girls given:

(a) The youngest child is a girl

(b) At least one is a girl.

Solution

Given that a family is having 2 children where both are girls.

Let us represent boy and the girl child with the letter (b) and (g) respectively.

Sample space = {(g, g), (g, b), (b, g), (b, b)}

Let us consider E be the event which indicates that both child born to a family are girls.

E = {(g, g)}

(a) Let us consider F be the event that the youngest child born in the family is a girl.

F = {(g, g), (b, g)}

E∩F=(gag)

P (F) = 2/4=1/2

P (E∩F) =1/4

P (E|F) =P(E∩F)/P(F)=1/2

(b) Let us consider G be the event that at least one child born in the family is a girl.

G = {(g, g), (b, g), (g, b)}

E∩G=(gag)

P(G) = 3/4

P (E∩G) =1/4

P (E|G) =P(E∩G)/P(G)=1/3

Question 13

An instructor has a question bank consisting of 300 easy True/False questions, 200 difficult True/False questions, 500 easy multiple choice questions and 400 difficult multiple choice questions. If a question is selected at random from the question bank, what is the probability that it will be an easy question given that it is a multiple choice? question?

Solution

	False / True	Multiple choice	Sum
Difficult	200	400	600
Easy	300	500	800
Sum	500	900	1400

Let us consider below events

Easy Questions = E

Difficult Questions = D

Multiple choice Questions = M

False / True Questions = T

Total Questions in the Question bank = 1400

Number of multiple choice Questions = 900

Number of False / True Questions = 500

Probability of getting an easy multiple choice Question in the Question bank =

P (E∩M) =500/1400=5/14

Probability of selecting a multiple-choice Questions be it easy or difficult

P (M) = 900/1400=914

The conditional probability of selecting an easy question given that it is a multiple choice question

= P (E|M) =P(E∩M)/ P(M)= (5/4) / (9/4) =59

Question 14

Given that the two numbers appearing on throwing the two dice are different. Find the probability of the event 'the sum of numbers on the dice is 4'.

Solutions

Total number of elements in the sample space = 36

Let Me be the event that the sum of two different numbers is 4 & F be the event that two numbers appearing on both the faces of the dice are different.

E = {(3, 1), (2, 2), (1, 3)}

F = {(6, 1), (6, 2), (6, 3), (6, 4), (6, 5), (6, 6)

(5, 1), (5, 2), (5, 3), (5, 4), (5, 5), (5, 6)

(4, 1), (4, 2), (4, 3), (4, 4), (4, 5), (4, 6)

(3, 1), (3, 2), (3, 3), (3, 4), (3, 5), (3, 6)

(2, 1), (2, 2), (2, 3), (2, 4), (2, 5), (2, 6)

(1, 2), (1, 3), (1, 4), (1, 5), (1, 6)}

E∩F= (1,3), (3,1))

P(F) = 30/36=5/6

E∩F=2/36=1/18

Then

$P(E|F) = P(E\cap F)/ P(F) = (1/18) / (5/6) = 1/15$

Question 15

Consider the experiment of throwing a die, if a multiple of 3 comes up, throw the die again and if any other number comes, toss a coin. Find the conditional probability of the event 'the coin shows a tail', given that 'at least one die shows a 3'.

Solution

Sample space of the above conducted experiment

= {(6, 6), (6, 5), (6, 4), (6, 3), (6, 2), (6, 1), (5, T), (5, H), (4, T), (4, H), (3, 6), (3, 5), (3, 4), (3, 3), (3, 2), (3, 1), (2, T), (2, H), (1, T), (1, H)}

Let Me be the event that a tail appears

F be the event at least one die shows 3

E = {(1, T), (2, T), (4, T), (5, T)}

F = {(3, 6), (3, 5), (3, 4), (3, 3), (3, 2), (3, 1), (6, 3)}

$E\cap F=\phi$
So, $P(E\cap F) = 0$

Then,

P (F) = {P (3, 6), P (3, 5), P (3, 4), P (3, 3), P (3, 2), P (3, 1), P (6, 3)}

= (1/6) (1/6) + (1/6) (1/6) + (1/6) (1/6) + (1/6) (1/6) + (1/6) (1/6) + (1/6) (1/6) + (1/6) (1/6) = 7/36

Probability that the die shows a tail given that at least one die shows 3

P (E|F) = 0/ (7/36) = 0

Question 16

If P (A) = 12 P (B) = 0,

Then P (A|B) =?

(a) 0

(b) 12

(c) not defined

(d) 1

Solution

It is given that:

P (A) = 12

P (B) = 0

P(A|B)

= P(A∩B)/ P(B)

= not defined

Hence, the correct answer is C

Question 17

If A and B are events such that P(A|B) = P(B|A), then:

(a) A⊂B, A ≠ B

(b) A = B

(c) A∩B=ϕ

(d) P (A) = P(B)

Solution

Given

P(A|B) = P(B|A)

P(A∩B)/P(B) = P(A∩B)/P(C)

P (A) = P(B)

The correct answer is D