

# Introduction to Trigonometry (revision sheet)

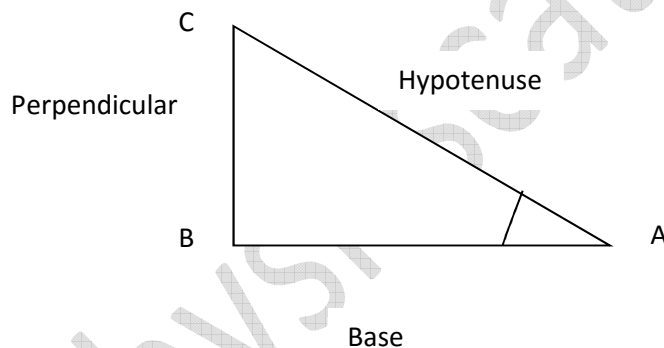
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**Trigonometry** (from Greek *trigōnon*, "triangle" and metron, "measure") is a branch of mathematics that studies relationships involving lengths and angles of triangles. The field emerged during the 3<sup>rd</sup> century BC from applications of geometry to astronomical studies.

Trigonometry is most simply associated with planar right angle triangles (each of which is a two-dimensional triangle with one angle equal to 90 degrees). The applicability to non-right-angle triangles exists, but, since any non-right-angle triangle (on a flat plane) can be bisected to create two right-angle triangles, most problems can be reduced to calculations on right-angle triangles. Thus the majority of applications relate to right-angle triangles

## Trigonometric Ratio's

In a right angle triangle ABC where  $B=90^\circ$ ,



We can define following term for angle A

**Base:** Side adjacent to angle

**Perpendicular:** Side Opposite of angle

**Hypotenuse:** Side opposite to right angle

We can define the trigonometric ratios for angle A as

$\sin A = \text{Perpendicular/Hypotenuse} = BC/AC$   
 $\operatorname{cosec} A = \text{Hypotenuse/Perpendicular} = AC/BC$   
 $\cos A = \text{Base/Hypotenuse} = AB/AC$   
 $\sec A = \text{Hypotenuse/Base} = AC/AB$   
 $\tan A = \text{Perpendicular/Base} = BC/AB$   
 $\cot A = \text{Base/Perpendicular} = AB/BC$

Notice that each ratio in the right-hand column is the inverse, or the reciprocal, of the ratio in the left-hand column.

The reciprocal of  $\sin A$  is  $\operatorname{cosec} A$ ; and vice-versa.

The reciprocal of  $\cos A$  is  $\sec A$

And the reciprocal of  $\tan A$  is  $\cot A$

These are valid for acute angles.

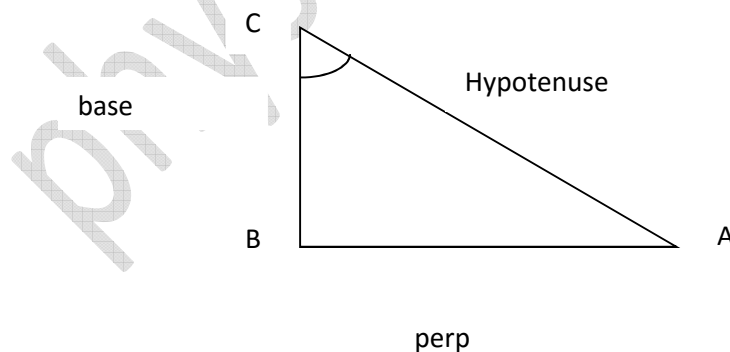
We can define  $\tan A = \sin A / \cos A$

And  $\cot A = \cos A / \sin A$

### Important Note

Since the hypotenuse is the longest side in a right triangle, the value of  $\sin A$  or  $\cos A$  is always less than 1 (or, in particular, equal to 1).

Similarly we can have define these for angle C



We can define the trigonometric ratios for angle C as

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$\sin C = \text{Perpendicular/Hypotenuse} = AB/AC$

$\operatorname{cosec} C = \text{Hypotenuse/Perpendicular} = AC/AB$

$\cos C = \text{Base/Hypotenuse} = BC/AC$

$\sec C = \text{Hypotenuse/Base} = AC/BC$

$\tan A = \text{Perpendicular/Base} = AB/BC$

$\cot A = \text{Base/Perpendicular} = BC/AB$

### Trigonometric Ratio's of Common angles

We can find the values of trigonometric ratio's various angle

Angles(A)	SinA	Cos A	TanA	Cosec A	Sec A	Cot A
$0^\circ$	0	1	0	Not defined	1	Not defined
$30^\circ$	$1/2$	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{3}}$	2	$\frac{2}{\sqrt{3}}$	$\sqrt{3}$
$45^\circ$	$\frac{1}{\sqrt{2}}$	$\frac{1}{\sqrt{2}}$	1	$\sqrt{2}$	$\sqrt{2}$	1
$60^\circ$	$\frac{\sqrt{3}}{2}$	$1/2$	$\sqrt{3}$	$\frac{2}{\sqrt{3}}$	2	$\frac{1}{\sqrt{3}}$
$90^\circ$	1	0	Not defined	1	Not defined	0

### Trigonometric ratio's of complimentary angles

We know that for Angle A, the complementary angle is  $90 - A$

In a right angle triangle ABC

$$A+B+C=180$$

$$\text{Now } B=90$$

So  $A + C = 90$

Or  $C = 90 - A$

We have seen in the previous section the value for trigonometric ratios for angle C

$\sin C = \frac{\text{Perpendicular}}{\text{Hypotenuse}} = \frac{AB}{AC}$

$\operatorname{cosec} C = \frac{\text{Hypotenuse}}{\text{Perpendicular}} = \frac{AC}{AB}$

$\cos C = \frac{\text{Base}}{\text{Hypotenuse}} = \frac{BC}{AC}$

$\sec C = \frac{\text{Hypotenuse}}{\text{Base}} = \frac{AC}{BC}$

$\tan C = \frac{\text{Perpendicular}}{\text{Base}} = \frac{AB}{BC}$

$\cot C = \frac{\text{Base}}{\text{Perpendicular}} = \frac{BC}{AB}$

This can be rewritten as

$\sin(90 - A) = \frac{AB}{AC}$

$\operatorname{cosec}(90 - A) = \frac{AC}{AB}$

$\cos(90 - A) = \frac{BC}{AC}$

$\sec(90 - A) = \frac{AC}{BC}$

$\tan(90 - A) = \frac{AB}{BC}$

$\cot(90 - A) = \frac{BC}{AB}$

Also we know that

$\sin A = \frac{BC}{AC}$

$\operatorname{cosec} A = \frac{AC}{BC}$

$\cos A = \frac{AB}{AC}$

$\sec A = \frac{AC}{AB}$

$\tan A = \frac{BC}{AB}$

$\cot A = \frac{AB}{BC}$

From both of these, we can easily make it out

**$\sin(90 - A) = \cos(A)$**

**$\cos(90 - A) = \sin A$**

$$\tan(90-A) = \cot A$$

$$\sec(90-A) = \operatorname{cosec} A$$

$$\operatorname{Cosec}(90-A) = \sec A$$

$$\cot(90-A) = \tan A$$

### Trigonometric identities

$$\sin^2 A + \cos^2 A = 1$$

$$1 + \tan^2 A = \sec^2 A$$

$$1 + \cot^2 A = \operatorname{cosec}^2 A$$