Trigonometry proof Questions

Question 1.

Given

a cosec A = p and b cot A = q,

then prove that $\frac{p^2}{a^2} - \frac{q^2}{b^2} = 1$.

Question 2.
Prove the following trigonometric identities:

a) $\cos^2\theta\,(1 + \tan^2\theta) = 1$

b) $\cos^2\theta + \frac{1}{1+cot2\,\theta} = 1$.

c) $\frac{1}{1+\sin\theta} + \frac{1}{1-\sin\theta} = 2\sec^2\theta$.

d) $\text{cosec}^2\theta + sec^2\,\theta = cosec^2\theta\,\sec^2\theta$.

e) $\tan\theta - \cot\theta = \frac{2\,sin2\,\theta - 1}{\sin\theta\cos\theta}$

Question 3.
If A, B, C are interior angles of Δ ABC, show that

$$\text{cosec}^2\left(\frac{B+C}{2}\right) - \tan^2\frac{A}{2} = 1.$$

Question 4.
Prove the following identities, where the angles involved are acute angles for which the expressions are defined.

a) $\frac{\cos A}{1+\sin A} + \frac{1+\sin A}{\cos A} = 2\sec A$

b) $\frac{1+\sec A}{\sec A} = \frac{sin2\,A}{1-\cos A}$

c) $\sqrt{\frac{1+\sin A}{1-\sin A}} = \sec A + \tan A$

Question 5.

If x = a sec $\theta + b\tan\theta$ *and* $y = a\tan\theta + b\sec\theta$, *prove that* $x^2 - y^2 = a^2 - b^2$.

Question 6.

Prove the following identities:

(i) $2(\sin^6\theta \;\; cos6\,\theta) - 3\,(sin4\,\theta + cos4\,\theta) + \; 1 = 0.$

(ii) $\sin^6\theta + cos6\,\theta + 3sin2\,\theta\; cos2\,\theta = 1$

(iii) $(\sin^8\theta - cos8\,\theta = (sin2\,\theta - cos2\,\theta)(1 - 2sin2\,\theta\; cos2\,\theta)$

Question 7.

If cosec A – cot A = q, then show that $\frac{q^2-1}{q^2+1} + \cos A = 0.$

Question 8.

If x = p sec α cos β, y = q sec α sin β and z = r tan α, then show that

$$\frac{x^2}{p^2} + \frac{y^2}{q^2} - \frac{z^2}{r^2} = 1.$$

Question 9.

Prove that

$(1 + \cot\theta)(1 + \tan\theta + \sec\theta) = 2$

Question 10.

Prove that

$\cot^4 A - 1 = \mathrm{cosec}^4 A - 2\mathrm{cosec}^2 A$