Coordinate Geometry Exercise 3

Question 1 Find the area of the triangle whose vertices are
(i) (2, 3), (−1, 0), (2, − 4) (ii) (−5, −1), (3, −5), (5, 2)
Question 2. In each of the following find the value of 'k', for which the points are collinear.
(i) (7, −2), (5, 1), (3, k) (ii) (8, 1), (k, − 4), (2, −5)
Question 3. Find the area of the triangle formed by joining the mid-points of the sides of the triangle whose vertices are (0, −1), (2, 1) and (0, 3). Find the ratio of this area to the area of the given triangle.
Question 4. Find the area of the quadrilateral whose vertices, taken in order, are (− 4, − 2), (− 3, − 5), (3, − 2) and (2, 3).
Question 5. You have studied in Class IX, (Chapter 9, Example 3), that a median of a triangle divides it into two triangles of equal areas. Verify this result for Δ ABC whose vertices are A (4, − 6), B (3, −2) and C (5, 2).

Solution 1

Area of triangle ABC of coordinates $A(x_1,y_1)$, $B(x_2,y_2)$ and $C(x_3,y_3)$

$$A = \frac{1}{2}[x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)]$$

i) Area of the given triangle = 1/2 [2 {0- (-4)} + (-1) {(-4) - (3)} + 2 (3 - 0)]

= 1/2 {8 + 7 + 6}
= 21/2 square units.
(ii) Area of the given triangle = 1/2 [-5 {(-5)- (4)} + 3(2-(-1)) + 5{-1 - (-5)}]
= 1/2{35 + 9 + 20}
= 32 square units

Solution2

Area of triangle ABC of coordinates $A(x_1,y_1)$, $B(x_2,y_2)$ and $C(x_3,y_3)$

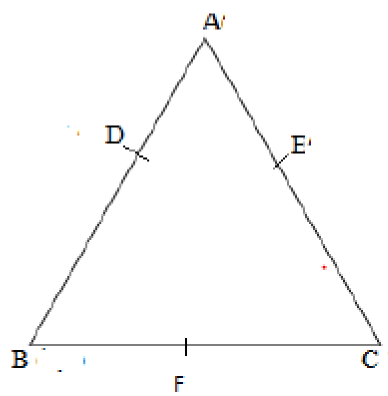
$$A = \frac{1}{2}[x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)]$$

For point A, B and C to be collinear, the value of A should be zero

(i) For collinear points, area of triangle formed by them is zero.

1/2 [7 {1- k} + 5(k-(-2)) + 3{(-2) + 1}] = 0
7 - 7k + 5k +10 -9 = 0
-2k + 8 = 0
k = 4
(ii) For collinear points, area of triangle formed by them is zero.
1/2 [8 { -4- (-5)} + k {(-5) -(1)} + 2{1 -(-4)}] = 0
8 - 6k + 10 = 0
6k = 18
k = 3

Solution 3



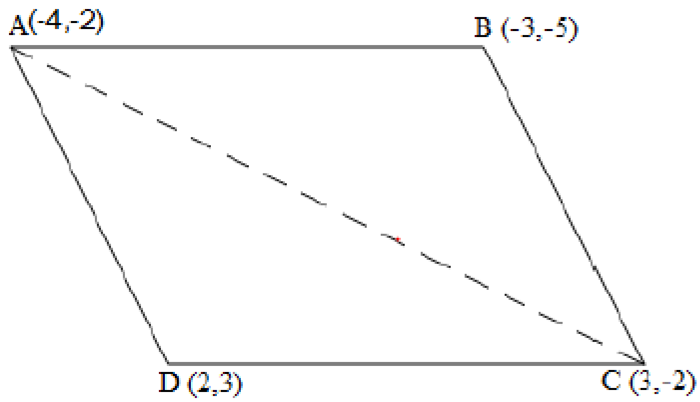
Let the vertices of the triangle be A (0, -1), B (2, 1), C (0, 3).
Let D, E, F be the mid-points of the sides of this triangle.
Coordinates of D, E, and F are given by mid-point formula
D = [(0+2)/2, (-1+1)/2] = (1,0)
E = [(0+0)/2, (-3-1)/2] = (0,1)
F = [(2+0)/2, (1+3)/2] = (1,2)
Now we know that

Area of triangle of coordinates A(x_1,y_1) , B(x_2,y_2) and C(x_3,y_3) is given by

$$A = \frac{1}{2}[x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)]$$

So Area of ΔDEF = 1/2 {1(2-1) + 1(1-0) + 0(0-2)}
= 1/2 (1+1) = 1 square units
Area of ΔABC = 1/2 [0(1-3) + 2{3-(-1)} + 0(-1-1)]
= 1/2 {8} = 4 square units
Therefore, the required ratio is 1:4.

Solution 4



Let the vertices of the quadrilateral be A (- 4, - 2), B (- 3, -5), C (3, - 2), and D (2, 3). Join AC to form two triangles

ΔABC and ΔACD.
Now we know that

Area of triangle of coordinates (x_1,y_1) , (x_2,y_2) and (x_3,y_3) is given by

$$A = \frac{1}{2}[x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)]$$

Area of ΔABC = 1/2 [(-4) {(-5) - (-2)} + (-3) {(-2) - (-2)} + 3{(-2) - (-5)}]
= 1/2 (12+0+9)
= 21/2 square units
Area of ΔACD = 1/2 [(-4) {(-2) - (3)} + 3{(3) - (-2)} + 2 {(-2) -(-2)}]
= 1/2 (20+15+0)
= 35/2 square units
Area of ΔABCD = Area of ΔABC + Area of ΔACD
= (21/2 + 35/2) square units = 28 square units

Solution 5

Let the vertices of the triangle be A (4, -6), B (3, -2), and C (5, 2).
Let D be the mid-point of side BC of ΔABC. Therefore, AD is the median in ΔABC.
Coordinates of point D (midpoint of B &C) = (3+5/2, -2+2/2) = (4,0)
Now we know that

Area of triangle of coordinates (x_1,y_1) , (x_2,y_2) and (x_3,y_3) is given by

$$A = \frac{1}{2}[x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)]$$

Area of ΔABD = 1/2 [(4) {(-2) - (0)} + 3{(0) - (-6)} + (4) {(-6) - (-2)}]
= 1/2 (-8+18-16)
= -3 square units
However, area cannot be negative. Therefore, area of ΔABD
is 3 square units.
Area of ΔADC = 1/2 [(4) {0 - (2)} + 4{(2) - (-6)} + (5) {(-6) -(0)}]
= 1/2 (-8+32-30)
= -3 square units
However, area cannot be negative. Therefore, area of ΔADC
is 3 square units.
The area of both sides is same. Thus, median AD has divided ΔABC in two triangles of equal areas