

Polynomial Exercise 2

Question 1

Find the zeroes of the following quadratic polynomials and verify the relationship between the zeroes and the coefficients.

(i) $x^2 - 2x - 8$

(ii) $4s^2 - 4s + 1$

(iii) $6x^2 - 3 - 7x$

(iv) $4u^2 + 8u$

(v) $t^2 - 15$

(vi) $3x^2 - x - 4$

Answer

(i) $x^2 - 2x - 8$

$$= x^2 - 4x + 2x - 8$$

$$= (x - 4)(x + 2)$$

Therefore, the zeroes of $x^2 - 2x - 8$ are 4 and -2.

(ii) $4s^2 - 4s + 1$

$$\text{From } (a-b)^2 = a^2 - 2ab + b^2$$

$$= (2s-1)^2$$

Therefore, the zeroes of $4s^2 - 4s + 1$ are $1/2$ and $1/2$.

(iii) $6x^2 - 3 - 7x$

$$= 6x^2 - 7x - 3$$

$$= 6x^2 - 9x + 2x - 3$$

$$= (3x + 1)(2x - 3)$$

Therefore, the zeroes of $6x^2 - 3 - 7x$ are $-1/3$ and $3/2$.

(iv) $4u^2 + 8u$

$$= 4u^2 + 8u$$

$$= 4u(u + 2)$$

Therefore, the zeroes of $4u^2 + 8u$ are 0 and -2.

(v) $t^2 - 15$

$$\text{From } (a^2 - b^2) = (a-b)(a+b)$$

$$= (t - \sqrt{15})(t + \sqrt{15})$$

Therefore, the zeroes of $t^2 - 15$ are $\sqrt{15}$ and $-\sqrt{15}$.

(vi) $3x^2 - x - 4$

$$= 3x^2 - 4x + 3x - 4$$

$$= (3x - 4)(x + 1)$$

Therefore, the zeroes of $3x^2 - x - 4$ are $4/3$ and -1 .

Verification of the relationship between the zeroes

S. No	Sum of zeroes = $-(\text{Coefficient of } x)/\text{Coefficient of } x^2$	Product of zeroes = $\text{Constant term}/\text{Coefficient of } x^2$.
i)	$4 + (-2) = 2 = -(-2)/1$	$4 \times (-2) = -8 = -8/1$
ii)	$1/2 + 1/2 = 1 = -(-4)/4$	$1/2 \times 1/2 = 1/4$
iii)	$-1/3 + 3/2 = 7/6 = -(-7)/6$	$-1/3 \times 3/2 = -1/2 = -3/6$
iv)	$0 + (-2) = -2 = -(8)/4$	$0 \times (-2) = 0 = 0/4$
v)	$\sqrt{15} + -\sqrt{15} = 0 = -0/1$	$(\sqrt{15})(-\sqrt{15}) = -15 = -15/1$
vi)	$4/3 + (-1) = 1/3 = -(-1)/3$	$4/3 \times (-1) = -4/3$

Question 2

Find a quadratic polynomial each with the given numbers as the sum and product of its zeroes respectively.

- (i) $1/4, -1$
- (ii) $\sqrt{2}, 1/3$
- (iii) $0, \sqrt{5}$
- (iv) $1, 1$
- (v) $-1/4, 1/4$
- (vi) $4, 1$

Answer

- (i) $1/4, -1$

Let the polynomial be $ax^2 + bx + c$, and its zeroes be p and q

$$p + q = 1/4 = -b/a$$

$$pq = -1 = -4/4 = c/a$$

$$\text{If } a = 4, \text{ then } b = -1, c = -4$$

Therefore, the quadratic polynomial is $4x^2 - x - 4$.

- (ii) $\sqrt{2}, 1/3$

Let the polynomial be $ax^2 + bx + c$, and its zeroes be p and q

$$p + q = \sqrt{2} = 3\sqrt{2}/3 = -b/a$$

$$pq = 1/3 = c/a$$

$$\text{If } a = 3, \text{ then } b = -3\sqrt{2}, c = 1$$

Therefore, the quadratic polynomial is $3x^2 - 3\sqrt{2}x + 1$.

(iii) $0, \sqrt{5}$

Let the polynomial be $ax^2 + bx + c$, and its zeroes be p and q

$$p + q = 0 = 0/1 = -b/a$$

$$pq = \sqrt{5} = \sqrt{5}/1 = c/a$$

$$\text{If } a = 1, \text{ then } b = 0, c = \sqrt{5}$$

Therefore, the quadratic polynomial is $x^2 + \sqrt{5}$.

(iv) $1, 1$

Let the polynomial be $ax^2 + bx + c$, and its zeroes be p and q

$$p + q = 1 = 1/1 = -b/a$$

$$pq = 1 = 1/1 = c/a$$

$$\text{If } a = 1, \text{ then } b = -1, c = 1$$

Therefore, the quadratic polynomial is $x^2 - x + 1$.

(v) $-1/4, 1/4$

Let the polynomial be $ax^2 + bx + c$, and its zeroes be p and q

$$p + q = -1/4 = -b/a$$

$$pq = 1/4 = c/a$$

$$\text{If } a = 4, \text{ then } b = 1, c = 1$$

Therefore, the quadratic polynomial is $4x^2 + x + 1$.

(vi) $4, 1$

Let the polynomial be $ax^2 + bx + c$, and its zeroes be p and q

$$p + q = 4 = 4/1 = -b/a$$

$$pq = 1 = 1/1 = c/a$$

$$\text{If } a = 1, \text{ then } b = -4, c = 1$$

Therefore, the quadratic polynomial is $x^2 - 4x + 1$.