NCERT Solutions for Polynomial Exercise 4

Question 1.

Verify that the numbers given alongside of the cubic polynomials below are their zeroes. Also verify the relationship between the zeroes and the coefficients in each case:
(i) $2x^3 + x^2 - 5x + 2$; 1/2, 1, -2
(ii) $x^3 - 4x^2 + 5x - 2$; 2, 1, 1

Answer

(i) $p(x) = 2x^3 + x^2 - 5x + 2$
Now for verification of zeroes, putting the given value in x.

$P(1/2) = 2(1/2)^3 + (1/2)^2 - 5(1/2) + 2$
= (2×1/8) + 1/4 - 5/2 + 2
= 1/4 + 1/4 - 5/2 + 2
= 1/2 - 5/2 + 2 = 0

$P(1) = 2(1)^3 + (1)^2 - 5(1) + 2$
= (2×1) + 1 - 5 + 2
= 2 + 1 - 5 + 2 = 0

$P(-2) = 2(-2)^3 + (-2)^2 - 5(-2) + 2$
= (2 × -8) + 4 + 10 + 2
= -16 + 16 = 0

Thus, 1/2, 1 and -2 are the zeroes of the given polynomial.

Comparing the given polynomial with $ax^3 + bx^2 + cx + d$, we get a=2, b=1, c=-5, d=2
Also, $k_1=1/2$, $k_2=1$ and $k_3=-2$

$$k_1 + k_2 + k_3 = -\frac{Coefficent\ of\ x^2}{Coefficent\ of\ x^3}$$

$$k_1 k_2 k_3 = -\frac{Contant\ term}{Coefficent\ of\ x^{32}}$$

$$k_1 k_2 + k_2 k_3 + k_1 k_3 = \frac{Coefficent\ of\ x}{Coefficent\ of\ x^2}$$

Now,
$-b/a = k_1 + k_2 + k_3$
⇒ 1/2 = 1/2 + 1 - 2
⇒ 1/2 = 1/2

$c/a = k_1k_2+k_2k_3+k_1k_3$
⇒ -5/2 = (1/2 × 1) + (1 × -2) + (-2 × 1/2)
⇒ -5/2 = 1/2 - 2 - 1
⇒ -5/2 = -5/2

$-d/a = k_1k_2k_3$
⇒ -2/2 = (1/2 × 1 × -2)
⇒ -1 = 1

Thus, the relationship between zeroes and the coefficients are verified.

(ii) $p(x) = x^3 - 4x^2 + 5x - 2$
Now for verification of zeroes, putting the given value in x.

$p(2) = 2^3 - 4(2)^2 + 5(2) - 2$
= 8 - 16 + 10 - 2
= 0

$p(1) = 1^3 - 4(1)^2 + 5(1) - 2$
= 1 - 4 + 5 - 2
= 0

$p(1) = 1^3 - 4(1)^2 + 5(1) - 2$
= 1 - 4 + 5 - 2
= 0

Thus, 2, 1 and 1 are the zeroes of the given polynomial.

Comparing the given polynomial with $ax^3 + bx^2 + cx + d$, we get a=1, b=-4, c=5, d=-2
Also, $k_1=2$, $k_2=1$ and $k_3=1$

$$k_1 + k_2 + k_3 = -\frac{Coefficent\ of\ x^2}{Coefficent\ of\ x^3}$$

$$k_1k_2k_3 = -\frac{Contant\ term}{Coefficent\ of\ x^{32}}$$

$$k_1k_2 + k_2k_3 + k_1k_3 = \frac{Coefficent\ of\ x}{Coefficent\ of\ x^2}$$

Now,
$-b/a = k_1 + k_2 + k_3$
⇒ 4/1 = 2 + 1 + 1

⇒ 4 = 4

$c/a = k_1k_2+k_2k_3+k_1k_3$
⇒ 5/1 = (2 × 1) + (1 × 1) + (1 × 2)
⇒ 5 = 2 + 1 + 2
⇒ 5 = 5

$-d/a = k_1k_2k_3$
⇒ 2/1 = (2 × 1 × 1)
⇒ 2 = 2

Thus, the relationship between zeroes and the coefficients are verified.

Question 2.

Find a cubic polynomial with the sum, sum of the product of its zeroes taken two at a time, and the product of its zeroes as 2, –7, –14 respectively.

Answer

Let the polynomial be $ax^3 + bx^2 + cx + d$ and the zeroes be k_1, k_2 and k_3

$$k_1 + k_2 + k_3 = -\frac{Coefficent\ of\ x^2}{Coefficent\ of\ x^3}$$

$$k_1k_2k_3 = -\frac{Contant\ term}{Coefficent\ of\ x^{32}}$$

$$k_1k_2 + k_2k_3 + k_1k_3 = \frac{Coefficent\ of\ x}{Coefficent\ of\ x^2}$$

Then, $k_1 + k_2 + k_3$ = -(-2)/1 = 2 = -b/a
$k_1k_2+k_2k_3+k_1k_3$= -7 = -7/1 = c/a
$k_1k_2k_3$ = -14 = -14/1 = -d/a

∴ a = 1, b = -2, c = -7 and d = 14
So, one cubic polynomial which satisfy the given conditions will be $x^3 - 2x^2 - 7x + 14$

Question 3.

If the zeroes of the polynomial $x^3 – 3x^2 + x + 1$ are a–b, a, a+b, find a and b.

Answer

Since, (a - b), a, (a + b) are the zeroes of the polynomial $x^3 – 3x^2 + x + 1$.

$$k_1 + k_2 + k_3 = -\frac{Coefficent\ of\ x^2}{Coefficent\ of\ x^3}$$

$$k_1 k_2 k_3 = -\frac{Contant\ term}{Coefficent\ of\ x^{32}}$$

$$k_1 k_2 + k_2 k_3 + k_1 k_3 = \frac{Coefficent\ of\ x}{Coefficent\ of\ x^2}$$

Therefore, sum of the zeroes = (a - b) + a + (a + b) = -(-3)/1 = 3

⇒ 3a = 3 ⇒ a =1

∴ Sum of the products of is zeroes taken two at a time = a(a - b) + a(a + b) + (a + b) (a - b) =1/1 = 1
$a^2 - ab + a^2 + ab + a^2 - b^2 = 1$
⇒ $3a^2 - b^2 = 1$

Putting the value of a,

⇒ $3(1)^2 - b^2 = 1$
⇒ $3 - b^2 = 1$
⇒ $b^2 = 2$
⇒ b = ±√2
Hence, a = 1 and b = ±√2

Question 4

If two zeroes of the polynomial $x^4 – 6x^3 – 26x^2 + 138x – 35$ are 2±√3, find other zeroes.

Answer

2+√3 and 2-√3 are two zeroes of the polynomial p(x) = $x^4 – 6x^3 – 26x^2 + 138x – 35$.
Let x = 2±√3
So, x-2 = ±√3
On squaring, we get $x^2 - 4x + 4 = 3$,
⇒ $x^2 - 4x + 1= 0$

Now, dividing p(x) by $x^2 - 4x + 1$

$$
\begin{array}{l|l}
 & x^2 - 2x - 35 \\
\hline
x^2 - 4x + 1 & x^4 - 6x^3 - 26x^2 + 138x - 35
\end{array}
$$

$x^4 \ -4x^3 \ +x^2$

$- \quad + \quad -$

$-2x^3 \ -27x^2 \ +138x \ -35$

$-2x^3 \ +8x^2 \ -2x$

$+ \quad - \quad +$

$-35x^2 \ +140x \ -35$

$-35x^2 \ +140x \ -35$

$+ \quad - \quad +$

0

$p(x) = x^4 - 6x^3 - 26x^2 + 138x - 35$

$= (x^2 - 4x + 1)(x^2 - 2x - 35)$

$= (x^2 - 4x + 1)(x^2 - 7x + 5x - 35)$

$= (x^2 - 4x + 1)[x(x - 7) + 5(x - 7)]$

$= (x^2 - 4x + 1)(x + 5)(x - 7)$

So $(x + 5)$ and $(x - 7)$ are other factors of $p(x)$.
Therefore

-5 and 7 are other zeroes of the given polynomial.

Question 5.
If the polynomial $x^4 - 6x^3 + 16x^2 - 25x + 10$ is divided by another polynomial $x^2 - 2x + k$, the remainder comes out to be $x + a$, find k and a.

Answer

On dividing $x^4 - 6x^3 + 16x^2 - 25x + 10$ by $x^2 - 2x + k$

$$x^2 - 2x + k \;\big)\; x^4 - 6x^3 + 16x^2 - 25x + 10 \;\big(\; x^2 - 4x + (8-k)$$

$$x^4 \;\; -2x^3 \;\; +kx^2$$
$$- \quad + \quad -$$
$$-4x^3 \;\; +(16-k)\,x^2 \;\; -25x \;\; +10$$

$$-4x^3 \;\; +8x^2 \;\; -4kx$$
$$+ \quad - \quad +$$
$$(8-k)x^2 \;\; +(4k-25)x \; +10$$

$$(8-k)\,x^2 \;\; -2(8-k)\,x \; +(8-k)\,k$$
$$- \quad + \quad -$$
$$(2k-9)\,x-(8-k)\,k+10$$

Remainder = (2k - 9)x - (8 - k)k + 10
But the remainder is given as x+ a.
On comparing their coefficients,
2k - 9 = 1
⇒ k = 10
⇒ k = 5 and,
-(8-k)k + 10 = a
⇒ a = -(8 - 5)5 + 10 =- 15 + 10 = -5
Hence, k = 5 and a = -5