Number system Exercise 1.5 and 1.6

Exercise 1.5

1. Classify the following numbers as rational or irrational:

(i) 2 - √5

(ii) (3 + √23) - √23

(iii) 2√7/7√7

(iv) 1/√2

(v) 2π

Answer

S.no	Number	Classification
i)	2 - √5 = 2 - 2.2360679… = - 0.2360679…	Since the number is non-terminating non-recurring therefore, it is an irrational number
ii)	(3 + √23) - √23 = 3 + √23 - √23 = 3 = 3/1	Since the number is rational number as it can be represented in *p/q* form
iii)	2√7/7√7 = 2/7	Since the number is rational number as it can be represented in *p/q* form.
iv)	1/√2 = √2/2 = 0.7071067811…	Since the number is non-terminating non-recurring therefore, it is an irrational number.
v)	2π = 2 × 3.1415… = 6.2830…	Since the number is non-terminating non-recurring therefore, it is an irrational number.

2. Simplify each of the following expressions:

(i) $(3 + \sqrt{3})(2 + \sqrt{2})$
(ii) $(3 + \sqrt{3})(3 - \sqrt{3})$
(iii) $(\sqrt{5} + \sqrt{2})^2$
(iv) $(\sqrt{5} - \sqrt{2})(\sqrt{5} + \sqrt{2})$

Answer

(i) $(3 + \sqrt{3})(2 + \sqrt{2})$
$= 3 \times 2 + 2 + \sqrt{3} + 3\sqrt{2} + \sqrt{3} \times \sqrt{2}$
$= 6 + 2\sqrt{3} + 3\sqrt{2} + \sqrt{6}$

(ii) $(3 + \sqrt{3})(3 - \sqrt{3})$
Now as $(a + b)(a - b) = a^2 - b^2$
$= 3^2 - (\sqrt{3})^2$
$= 9 - 3 = 6$

(iii) $(\sqrt{5} + \sqrt{2})^2$
Now as $(a + b)^2 = a^2 + b^2 + 2ab$
$= (\sqrt{5})^2 + (\sqrt{2})^2 + 2 \times \sqrt{5} \times \sqrt{2}$
$= 5 + 2 + 2 \times \sqrt{5} \times 2$
$= 7 + 2\sqrt{10}$

(iv) $(\sqrt{5} - \sqrt{2})(\sqrt{5} + \sqrt{2})$
Now as $(a + b)(a - b) = a^2 - b^2$
$= (\sqrt{5})^2 - (\sqrt{2})^2$
$= 5 – 2 = 3$

3. Recall, π is defined as the ratio of the circumference (say c) of a circle to its diameter (say d). That is, π = c/d. This seems to contradict the fact that π is irrational. How will you resolve this contradiction?

Answer

There is no contradiction. When we measure a value with a scale, we only obtain an approximate value. We never obtain an exact value. Therefore, we may not realize that either c or d is irrational. The value of π is almost equal to 22/7 or 3.142857...

4. Represent $\sqrt{9.3}$ on the number line.

5. Rationalize the denominators of the following:
(i) 1/√7
(ii) 1/ (√7-√6)
(iii) 1/ (√5+√2)
(iv) 1/ (√7-2)

Answer

i) $\frac{1}{\sqrt{7}} = \frac{1\times\sqrt{7}}{\sqrt{7}\times\sqrt{7}} = \frac{\sqrt{7}}{7}$

ii) $\frac{1}{\sqrt{7}-\sqrt{6}} = \frac{1(\sqrt{7}+\sqrt{6})}{(\sqrt{7}-\sqrt{6})(\sqrt{7}+\sqrt{6})} = \frac{\sqrt{7}+\sqrt{6}}{1}$

iii)) $\frac{1}{\sqrt{5}+\sqrt{2}} = \frac{1(\sqrt{5}-\sqrt{2})}{(\sqrt{5}+\sqrt{2})(\sqrt{5}-\sqrt{2})} = \frac{\sqrt{5}-\sqrt{2}}{3}$

iv) $\frac{1}{\sqrt{7}-2} = \frac{1(\sqrt{7}+2)}{(\sqrt{7}-2)(\sqrt{7}+2)} = \frac{\sqrt{7}+2}{3}$

Exercise 1.6

1. Find:
(i) $64^{1/2}$
(ii) $32^{1/5}$
(iii) $125^{1/3}$

Answer

i) $64^{1/2} = (2^6)^{1/2} = 2^{6 \times 1/2} = 2^3 = 8$

ii) $32^{1/5} = (2^5)^{1/5} = 2$

iii) $125^{1/3} = (5^3)^{1/3} = 5$

2. Find:

(i) $9^{3/2}$
(ii) $32^{2/5}$
(iii) $16^{3/4}$
(iv) $125^{-1/3}$

Answer

i) $9^{3/2} = (3^2)^{3/2} = 27$

ii) $32^{2/5} = (2^5)^{2/5} = 4$
iii) $16^{3/4} = (2^4)^{3/4} = 8$
iv) $125^{-1/3} = 1/125^{1/3} = 1/ (5^3)^{1/3} = 1/5$

3. Simplify:

(i) $2^{2/3}.2^{1/5}$
(ii) $(1/3^3)^7$
(iii) $11^{1/2}/11^{1/4}$
(iv) $7^{1/2}.8^{1/2}$

Answer

i) $2^{2/3}.2^{1/5} = 2^{2/3+ 1/5} = 2^{10+3/15} = 2^{13/15}$

ii) $(1/3^3)^7$

$=1/3^{3X7} = 1/3^{21} = 3^{-21}$

iii) $11^{1/2}/11^{1/4}$
$= 11^{1/2 -1/4} = 11^{1/4}$

iv) $7^{1/2}.8^{1/2}$
$=(7X8)^{1/2} = 56^{1/2}$