Polynomial Exercise -2.4 and 2.5

Exercise 2.4

Question 1

Determine which of the following polynomials has (x + 1) a factor:

(i) $x^3 + x^2 + x + 1$

(ii) $x^4 + x^3 + x^2 + x + 1$

(iii) $x^4 + 3x^3 + 3x^2 + x + 1$

(iv) $x^3 - x^2 - (2 + \sqrt{2})x + \sqrt{2}$

Solution

We know from remainder theorem, if (x-a) is a factor of polynomial p(x) then p(a)=0

(i) If (x + 1) is a factor of $p(x) = x^3 + x^2 + x + 1$

Then p (-1) must be zero.
Here, $p(x) = x^3 + x^2 + x + 1$
$p(-1) = (-1)^3 + (-1)^2 + (-1) + 1$
$= -1 + 1 - 1 + 1 = 0$
Hence, x + 1 is a factor of this polynomial

(ii) If (x + 1) is a factor of $p(x) = x^4 + x^3 + x^2 + x + 1$

Then p (-1) must be zero.
Here, $p(x) = x^4 + x^3 + x^2 + x + 1$
$p(-1) = (-1)^4 + (-1)^3 + (-1)^2 + (-1) + 1$
$= 1$

As, $p(-1) \neq 0$
Hence, x + 1 is not a factor of this polynomial
(iii)If (x + 1) is a factor of polynomial $p(x) = x^4 + 3x^3 + 3x^2 + x + 1$

Then p (- 1) must be 0.
$p(-1) = (-1)^4 + 3(-1)^3 + 3(-1)^2 + (-1) + 1$
$= 1$
As, $p(-1) \neq 0$
Hence, x + 1 is not a factor of this polynomial.

(iv) If $(x + 1)$ is a factor of polynomial
$p(x) = x^3 - x^2 - (2 + \sqrt{2})x + \sqrt{2}$
Then $p(-1)$ must be 0.
$p(-1) = (-1)^3 - (-1)^2 - (2 + \sqrt{2})(-1) + \sqrt{2}$
$= -1 - 1 + 2 + \sqrt{2} + \sqrt{2}$
$= 2\sqrt{2}$
As, $p(-1) \neq 0$
Hence, $x + 1$ is not a factor of this polynomial.

Question 2

Use the Factor Theorem to determine whether $g(x)$ is a factor of $p(x)$ in each of the following cases:
(i) $p(x) = 2x^3 + x^2 - 2x - 1$, $g(x) = x + 1$
(ii) $p(x) = x^3 + 3x^2 + 3x + 1$, $g(x) = x + 2$
(iii) $p(x) = x^3 - 4x^2 + x + 6$, $g(x) = x - 3$

Solution

(i) If $g(x) = x + 1$ is a factor of given polynomial $p(x)$

Then, $p(-1)$ must be zero.
$p(x) = 2x^3 + x^2 - 2x - 1$
$p(-1) = 2(-1)^3 + (-1)^2 - 2(-1) - 1$
$= 2(-1) + 1 + 2 - 1 = 0$
Hence, $g(x) = x + 1$ is a factor of given polynomial.

(ii) If $g(x) = x + 2$ is a factor of given polynomial $p(x)$

Then, $p(-2)$ must be 0.
$p(x) = x^3 + 3x^2 + 3x + 1$
$p(-2) = (-2)^3 + 3(-2)^2 + 3(-2) + 1$
$= -8 + 12 - 6 + 1$
$= -1$

As, $p(-2) \neq 0$
Hence $g(x) = x + 2$ is not a factor of given polynomial.

(iii) If $g(x) = x - 3$ is a factor of given polynomial $p(x)$
Then, $p(3)$ must be 0.
$p(x) = x^3 - 4x^2 + x + 6$
$p(3) = (3)^3 - 4(3)^2 + 3 + 6$
$= 27 - 36 + 9 = 0$
Hence, $g(x) = x - 3$ is a factor of given polynomial.

Question 3

Find the value of k, if x - 1 is a factor of p(x) in each of the following cases:

(i) $p(x) = x^2 + x + k$
(ii) $p(x) = 2x^2 + kx + \sqrt{2}$
(iii) $p(x) = kx^2 - \sqrt{2}x + 1$
(iv) $p(x) = kx^2 - 3x + k$

Solution

(i) If x - 1 is a factor of polynomial $p(x) = x^2 + x + k$, then

$p(1) = 0$
$(1)^2 + 1 + k = 0$
$2 + k = 0$
$k = -2$
So, value of k is -2.

(ii) If x - 1 is a factor of polynomial $p(x) = 2x^2 + kx + \sqrt{2}$, then
$p(1) = 0$
$2(1)^2 + k(1) + \sqrt{2} = 0$
$k = -2 - \sqrt{2} = -(2 + \sqrt{2})$
So, value of k is $-(2 + \sqrt{2})$.

(iii) If x - 1 is a factor of polynomial $p(x) = kx^2 - \sqrt{2}x + 1$, then
$p(1) = 0$
$k(1)^2 - \sqrt{2}(1) + 1 = 0$
$k = \sqrt{2} - 1$
So, value of k is $\sqrt{2} - 1$.

(iv) If x - 1 is a factor of polynomial $p(x) = kx^2 - 3x + k$, then
$p(1) = 0$
$k(1)^2 + 3(1) + k = 0$
$k - 3 + k = 0$
$k = 3/2$
So, value of k is 3/2.

Question 4
Factorise:
(i) $12x^2 + 7x + 1$
(ii) $2x^2 + 7x + 3$
(iii) $6x^2 + 5x - 6$
(iv) $3x^2 - x - 4$

Solution

Here we would be using splitting the middle term to factorize the polynomial

To factorise $ax^2 + bx + c$, we should write b as the sum of two numbers whose product is ac

(i) $12x^2 + 7x + 1$

Here a=12, c=1 and b=7 So 7=3+4 ,3X4=12X1=12
$= 12x^2 + 4x + 3x + 1$
$= 4x (3x + 1) + 1 (3x + 1)$
$= (3x + 1) (4x + 1)$

(ii) $2x^2 + 7x + 3$

Here a=2, c=3 and b=7 So b=7=6+1 ,6X1=2X3=6

$= 2x^2 + 6x + x + 3$
$= 2x (x + 3) + 1 (x + 3)$
$= (x + 3) (2x + 1)$

(iii) $6x^2 + 5x - 6$

Here a=6, c=-6 and b=5 So b=5=9+(-4) ,9X (-4) =6X (-6) =-36

$= 6x^2 + 9x - 4x - 6$

$= 3x (2x + 3) - 2 (2x + 3)$
$= (2x + 3) (3x - 2)$

(iv) $3x^2 - x - 4$
Here a=3, c=-4 and b=-1 So b=-1=-4+3, (-)4X3=3X (-4) =-12
$= 3x^2 - 4x + 3x - 4$
$= x (3x - 4) + 1 (3x - 4)$
$= (3x - 4) (x + 1)$

Question 5

Factorise:
(i) $x^3 - 2x^2 - x + 2$

(ii) $x^3 - 3x^2 - 9x - 5$
(iii) $x^3 + 13x^2 + 32x + 20$
(iv) $2y^3 + y^2 - 2y - 1$

Solution

(i) Let p(x) = $x^3 - 2x^2 - x + 2$
Factors of 2 are ±1 and ± 2
By trial method, we find that
p (-1) = 0
So, (x+1) is factor of p(x)

	$x^2 - 3x + 2$
x+1	$x^3 - 2x^2 - x + 2$ $x^3 + x^2$ - -
	$-3x^2 - x + 2$ $-3x^2 - 3x$ + +
	2x+2 2x +2 - -
	0

Now, Dividend = Divisor × Quotient + Remainder
(x+1) (x^2 - 3x + 2)
Factorizing the second part by split middle term method
= (x+1) (x^2 - x - 2x + 2)
= (x+1) [x(x-1) -2(x-1)]
= (x+1) (x-1) (x+2)

(ii) Let p(x) = $x^3 - 3x^2 - 9x - 5$
Factors of 5 are ±1 and ±5
By trial method, we find that
p (5) = 0
So, (x-5) is factor of p(x)

	$x^2 + 2x + 1$
x-5	$x^3 - 3x^2 - 9x - 5$ $x^3 - 5x^2$ - +
	$2x^2 - 9x - 5$ $2x^2 - 10x$ - +
	$x - 5$ $x - 5$ - +
	0

Now, Dividend = Divisor × Quotient + Remainder

$(x-5)(x^2 + 2x + 1)$

Factorizing the second part by split middle term method

$= (x-5)(x^2 + x + x + 1)$

$= (x-5)\{x(x+1) + 1(x+1)\}$

$= (x-5)(x+1)(x+1)$

(iii) Let $p(x) = x^3 + 13x^2 + 32x + 20$

Factors of 20 are ±1, ±2, ±4, ±5, ±10 and ±20

By trial method, we find that

$p(-1) = 0$

So, $(x+1)$ is factor of $p(x)$

	$x^2 + 12x + 20$
x+1	$x^3 + 13x^2 + 32x + 20$ $x^3 + x^2$ - -
	$12x^2 + 32x + 20$ $12x^2 + 12x$ - -
	$20x + 20$ $20x + 20$ - -
	0

Now, Dividend = Divisor × Quotient + Remainder

$(x+1)(x^2 + 12x + 20)$

Factorizing the second part by split middle term method

$= (x+1)(x^2 + 2x + 10x + 20)$

$= (x-5)[x(x+2) + 10(x+2)]$

$= (x-5)(x+2)(x+10)$

(iv) Let $p(y) = 2y^3 + y^2 - 2y - 1$
Factors of ab = 2× (-1) = -2 are ±1 and ±2
By trial method, we find that
p (1) = 0
So, (y-1) is factor of p(y)

	$2y^2 + 3y + 1$
$y-1$	$2y^3 + y^2 - 2y - 1$ $2y^3 - 2y^2$ $-\quad +$
	$3y^2 - 2y - 1$ $3y^2 - 3y$ $-\quad +$
	$y - 1$ $y - 1$ $-\quad +$
	0

Now, Dividend = Divisor × Quotient + Remainder
$(y-1)(2y^2 + 3y + 1)$
Factorizing the second part by split middle term method
$= (y-1)(2y^2 + 2y + y + 1)$
$= (y-1)[2y(y+1) + 1(y+1)]$
$= (y-1)(2y+1)(y+1)$

Exercise 2.5

Question 1

Use suitable identities to find the following products:
(i) $(x + 4)(x + 10)$

(ii) $(x + 8)(x - 10)$

(iii) $(3x + 4)(3x - 5)$
(iv) $(y^2 + 3/2)(y^2 - 3/2)$

(v) $(3 - 2x)(3 + 2x)$

Solution

(i) Using identity, $(x + a)(x + b) = x^2 + (a + b)x + ab$
Here $a = 4$ and $b = 10$
Therefore,
$(x + 4)(x + 10) = x^2 + (4 + 10)x + (4 \times 10)$
$= x^2 + 14x + 40$

(ii) $(x + 8)(x - 10)$
Using identity, $(x + a)(x + b) = x^2 + (a + b)x + ab$
Here, $a = 8$ and $b = -10$

Therefore,
$(x + 8)(x - 10) = x^2 + \{8 + (-10)\}x + \{8 \times (-10)\}$
$= x^2 + (8 - 10)x - 80$
$= x^2 - 2x - 80$

(iii) $(3x + 4)(3x - 5)$
Using identity, $(x + a)(x + b) = x^2 + (a + b)x + ab$
Here, $x = 3x$, $a = 4$ and $b = -5$

Therefore,
$(3x + 4)(3x - 5) = (3x)^2 + \{4 + (-5)\}3x + \{4 \times (-5)\}$
$= 9x^2 + 3x(4 - 5) - 20$
$= 9x^2 - 3x - 20$

(iv) $(y^2 + 3/2)(y^2 - 3/2)$
Using identity, $(x + y)(x - y) = x^2 - y^2$
Here, $x = y^2$ and $y = 3/2$

Therefore,
$(y^2 + 3/2)(y^2 - 3/2) = (y^2)^2 - (3/2)^2$
$= y^4 - 9/4$

(v) $(3 - 2x)(3 + 2x)$
Using identity, $(x + y)(x - y) = x^2 - y^2$
Here, $x = 3$ and $y = 2x$

Therefore,
$(3 - 2x)(3 + 2x) = 3^2 - (2x)^2$
$= 9 - 4x^2$

Question 2

Evaluate the following products without multiplying directly:
(i) 103×107

(ii) 95×96

(iii) 104×96

Solution

(i) 103×107 can be written as

$= (100 + 3)(100 + 7)$
Using identity, $(x + a)(x + b) = x^2 + (a + b)x + ab$

Here, $x = 100$, $a = 3$ and $b = 7$
Therefore,
$103 \times 107 = (100 + 3)(100 + 7) = (100)^2 + (3 + 7)10 + (3 \times 7)$
$= 10000 + 100 + 21$
$= 10121$
(ii) $95 \times 96 = (90 + 5)(90 + 4)$
Using identity, $(x + a)(x + b) = x^2 + (a + b)x + ab$
Here, $x = 90$, $a = 5$ and $b = 4$

Therefore,

$95 \times 96 = (90 + 5)(90 + 4) = 90^2 + 90(5 + 6) + (5 \times 6)$
$= 8100 + (11 \times 90) + 30$
$= 8100 + 990 + 30 = 9120$

(iii) $104 \times 96 = (100 + 4)(100 - 4)$
Using identity, $(x + y)(x - y) = x^2 - y^2$
Here, $x = 100$ and $y = 4$
Therefore,

$104 \times 96 = (100 + 4)(100 - 4) = (100)^2 - (4)^2 = 10000 - 16 = 9984$

Question 3

Factorise the following using appropriate identities:
(i) $9x^2 + 6xy + y^2$

(ii) $4y^2 - 4y + 1$

(iii) $x^2 - y^2/100$

Solution

(i) $9x^2 + 6xy + y^2 = (3x)^2 + (2 \times 3x \times y) + y^2$
Using identity, $(a + b)^2 = a^2 + 2ab + b^2$

Here, $a = 3x$ and $b = y$

$9x^2 + 6xy + y^2 = (3x)^2 + (2\times 3x\times y) + y^2 = (3x + y)^2 = (3x + y)(3x + y)$

(ii) $4y^2 - 4y + 1 = (2y)^2 - (2\times 2y\times 1) + 1^2$

Using identity, $(a - b)^2 = a^2 - 2ab + b^2$

Here, $a = 2y$ and $b = 1$

$4y^2 - 4y + 1 = (2y)^2 - (2\times 2y\times 1) + 1^2 = (2y - 1)^2 = (2y - 1)(2y - 1)$

(iii) $x^2 - y^2/100 = x^2 - (y/10)^2$

Using identity, $a^2 - b^2 = (a + b)(a - b)$

Here, $a = x$ and $b = (y/10)$

$x^2 - y^2/100 = x^2 - (y/10)^2 = (x - y/10)(x + y/10)$

Question 4

Expand each of the following, using suitable identities:

(i) $(x + 2y + 4z)^2$

(ii) $(2x - y + z)^2$

(iii) $(-2x + 3y + 2z)^2$

(iv) $(3a - 7b - c)^2$

(v) $(-2x + 5y - 3z)^2$

(vi) $[1/4\ a - 1/2\ b + 1]^2$

Answer

<u>We will be using the identity</u>

$(a + b + c)^2 = a^2 + b^2 + c^2 + 2ab + 2bc + 2ca$

(i) $(x + 2y + 4z)^2$

Here, $a = x$, $b = 2y$ and $c = 4z$

$(x + 2y + 4z)^2 = x^2 + (2y)^2 + (4z)^2 + (2\times x\times 2y) + (2\times 2y\times 4z) + (2\times 4z\times x)$

$= x^2 + 4y^2 + 16z^2 + 4xy + 16yz + 8xz$

(ii) $(2x - y + z)^2$

Here, $a = 2x$, $b = -y$ and $c = z$

$(2x - y + z)^2 = (2x)^2 + (-y)^2 + z^2 + (2\times 2x\times -y) + (2\times -y\times z) + (2\times z\times 2x)$

$= 4x^2 + y^2 + z^2 - 4xy - 2yz + 4xz$

(iii) $(-2x + 3y + 2z)^2$

Here, $a = -2x$, $b = 3y$ and $c = 2z$

$(−2x + 3y + 2z)^2 = (-2x)^2 + (3y)^2 + (2z)^2 + (2×-2x×3y) + (2×3y×2z) + (2×2z×-2x)$

$= 4x^2 + 9y^2 + 4z^2 - 12xy + 12yz - 8xz$

(iv) $(3a − 7b − c)^2$

Here, a = 3a, b = -7band c = -c

$(3a − 7b − c)^2 = (3a)^2 + (-7b)^2 + (-c)^2 + (2×3a×-7b) + (2×-7b×-c) + (2×-c×3a)$

$= 9a^2 + 49b^2 + c^2 - 42ab + 14bc - 6ac$

(v) $(−2x + 5y − 3z)^2$

Here, a = -2x, b = 5y and c = -3z

$(−2x + 5y − 3z)^2 = (-2x)^2 + (5y)^2 + (-3z)^2 + (2×-2x×5y) + (2×5y×-3z) + (2×-3z×-2x)$

$= 4x^2 + 25y^2 + 9z^2 - 20xy - 30yz + 12xz$

(vi) $[1/4\ a - 1/2\ b + 1]^2$

Here, a = 1/4 a, b = -1/2 band c = 1

$[1/4\ a - 1/2\ b + 1]^2 = (1/4\ a)^2 + (-1/2\ b)^2 + 1^2 + (2×1/4\ a×-1/2\ b) + (2×-1/2\ b×1) + (2×1×1/4\ a)$

$= 1/16\ a^2 + 1/4\ b^2 + 1 - 1/4\ ab - b + 1/2\ a$

Question 5

Factorise:

(i) $4x^2 + 9y^2 + 16z^2 + 12xy - 24yz - 16xz$

(ii) $2x^2 + y^2 + 8z^2 - 2\sqrt{2}\ xy + 4\sqrt{2}\ yz - 8xz$

Solution

(i) $4x^2 + 9y^2 + 16z^2 + 12xy - 24yz - 16xz$

Using identity, $(a + b + c)^2 = a^2 + b^2 + c^2 + 2ab + 2bc + 2ca$

$4x^2 + 9y^2 + 16z^2 + 12xy - 24yz - 16xz$

$= (2x)^2 + (3y)^2 + (-4z)^2 + (2×2x×3y) + (2×3y×-4z) + (2×-4z×2x)$

$= (2x + 3y - 4z)^2$

$= (2x + 3y - 4z)(2x + 3y - 4z)$

(ii) $2x^2 + y^2 + 8z^2 - 2\sqrt{2}\ xy + 4\sqrt{2}\ yz - 8xz$

Using identity, $(a + b + c)^2 = a^2 + b^2 + c^2 + 2ab + 2bc + 2ca$

$2x^2 + y^2 + 8z^2 - 2\sqrt{2}\ xy + 4\sqrt{2}\ yz - 8xz$

$= (-\sqrt{2}x)^2 + (y)^2 + (2\sqrt{2}z)^2 + (2×-\sqrt{2}x×y) + (2×y×2\sqrt{2}z) + (2×2\sqrt{2}z×-\sqrt{2}x)$

$= (-\sqrt{2}x + y + 2\sqrt{2}z)^2$

$= (-\sqrt{2}x + y + 2\sqrt{2}z)(-\sqrt{2}x + y + 2\sqrt{2}z)$

Question 6

Write the following cubes in expanded form:
(i) $(2x + 1)^3$

(ii) $(2a - 3b)^3$

(iii) $[3/2\ x + 1]^3$

(iv) $[x - 2/3\ y]^3$

Solution

We will be using the identity

$(a + b)^3 = a^3 + b^3 + 3ab\ (a + b)$

$(a - b)^3 = a^3 - b^3 - 3ab\ (a - b)$

(i) $(2x + 1)^3$
$=(2x)^3 + 1^3 + (3\times2x\times1)\ (2x + 1)$
$= 8x^3 + 1 + 6x\ (2x + 1)$
$= 8x^3 + 12x^2 + 6x + 1$

(ii) $(2a - 3b)^3$
$= (2a)^3 - (3b)^3 - (3\times2a\times3b)\ (2a - 3b)$
$= 8a^3 - 27b^3 - 18ab\ (2a - 3b)$
$= 8a^3 - 27b^3 - 36a^2b + 54ab^2$

(iii) $[3/2\ x + 1]^3$
$= (3/2\ x)^3 + 1^3 + (3\times3/2\ x\times1)\ (3/2\ x + 1)$
$= 27/8\ x^3 + 1 + 9/2\ x\ (3/2\ x + 1)$
$= 27/8\ x^3 + 1 + 27/4\ x^2 + 9/2\ x$
$= 27/8\ x^3 + 27/4\ x^2 + 9/2\ x + 1$

(iv) $[x - 2/3\ y]^3 = (x)^3 - (2/3\ y)^3 - (3\times x\times2/3\ y)\ (x - 2/3\ y)$
$= x^3 - 8/27y^3 - 2xy\ (x - 2/3\ y)$
$= x^3 - 8/27y^3 - 2x^2y + 4/3xy^2$

Question 7

Find the value of the following using suitable identities:
(i) $(99)^3$

(ii) $(102)^3$

(iii) $(998)^3$

Answer

<u>We will be using the identity</u>

$(a + b)^3 = a^3 + b^3 + 3ab (a + b)$
$(a - b)^3 = a^3 - b^3 - 3ab (a - b)$

(i) $(99)^3 = (100 - 1)^3$
$= (100)^3 - 1^3 - (3\times100\times1) (100 - 1)$
$= 1000000 - 1 - 300(100 - 1)$
$= 1000000 - 1 - 30000 + 300$
$= 970299$

(ii) $(102)^3 = (100 + 2)^3$
$= (100)^3 + 2^3 + (3\times100\times2) (100 + 2)$
$= 1000000 + 8 + 600(100 + 2)$
$= 1000000 + 8 + 60000 + 1200$
$= 1061208$

(iii) $(998)^3$
$= (1000)^3 - 2^3 - (3\times1000\times2) (1000 - 2)$
$= 100000000 - 8 - 6000(1000 - 2)$
$= 100000000 - 8- 600000 + 12000$
$= 994011992$

Question 8

Factorise each of the following:
(i) $8a^3 + b^3 + 12a^2b + 6ab^2$

(ii) $8a^3 - b^3 - 12a^2b + 6ab^2$
(iii) $27 - 125a^3 - 135a + 225a^2$

(iv) $64a^3 - 27b^3 - 144a^2b + 108ab^2$
(v) $27p^3 - 1/216 - 9/2\ p^2 + 1/4\ p$

Solution

(i) $8a^3 + b^3 + 12a^2b + 6ab^2$
Using identity, $(a + b)^3 = a^3 + b^3 + 3a^2b + 3ab^2$
$8a^3 + b^3 + 12a^2b + 6ab^2$
$= (2a)^3 + b^3 + 3(2a)^2b + 3(2a) (b)^2$
$= (2a + b)^3$
$= (2a + b) (2a + b) (2a + b)$

(ii) $8a^3 - b^3 - 12a^2b + 6ab^2$
Using identity, $(a - b)^3 = a^3 - b^3 - 3a^2b + 3ab^2$
$8a^3 - b^3 - 12a^2b + 6ab^2 = (2a)^3 - b^3 - 3(2a)^2b + 3(2a)(b)^2$
$= (2a - b)^3$
$= (2a - b)(2a - b)(2a - b)$

(iii) $27 - 125a^3 - 135a + 225a^2$
Using identity, $(a - b)^3 = a^3 - b^3 - 3a^2b + 3ab^2$
$27 - 125a^3 - 135a + 225a^2 = 3^3 - (5a)^3 - 3(3)^2(5a) + 3(3)(5a)^2$
$= (3 - 5a)^3$
$= (3 - 5a)(3 - 5a)(3 - 5a)$

(iv) $64a^3 - 27b^3 - 144a^2b + 108ab^2$
Using identity, $(a - b)^3 = a^3 - b^3 - 3a^2b + 3ab^2$
$64a^3 - 27b^3 - 144a^2b + 108ab^2 = (4a)^3 - (3b)^3 - 3(4a)^2(3b) + 3(4a)(3b)^2$
$= (4a - 3b)^3$
$= (4a - 3b)(4a - 3b)(4a - 3b)$

(v) $27p^3 - 1/216 - 9/2\, p^2 + 1/4\, p$

Using identity, $(a - b)^3 = a^3 - b^3 - 3a^2b + 3ab^2$
$27p^3 - 1/216 - 9/2\, p^2 + 1/4\, p$
$= (3p)^3 - (1/6)^3 - 3(3p)^2(1/6) + 3(3p)(1/6)^2$
$= (3p - 1/6)^3$
$= (3p - 1/6)(3p - 1/6)(3p - 1/6)$

Question 9

Verify the below identities

(i) $x^3 + y^3 = (x + y)(x^2 - xy + y^2)$

(ii) $x^3 - y^3 = (x - y)(x^2 + xy + y^2)$

Solution

(i) $x^3 + y^3 = (x + y)(x^2 - xy + y^2)$
We know that,
$(x + y)^3 = x^3 + y^3 + 3xy(x + y)$

Taking $3xy(x + y)$ on another side, we get
$x^3 + y^3 = (x + y)^3 - 3xy(x + y)$

Taking $(x + y)$ common
$= (x + y)[(x + y)^2 - 3xy]$

= $(x + y)[(x^2 + y^2 + 2xy) -3xy]$
= $(x + y)(x^2 + y^2 - xy)$

(ii) $x^3 - y^3 = (x - y) (x^2 + xy + y^2)$
We know that,
$(x - y)^3 = x^3 - y^3 - 3xy(x - y)$
$x^3 - y^3 = (x - y)^3 +3xy(x - y)$

Taking (x-y) common
= $(x - y)[(x - y)^2 +3xy]$
= $(x - y)[(x^2 + y^2 - 2xy) +3xy]$
= $(x + y)(x^2 + y^2 + xy)$

Question 10

Factorise each of the following:
(i) $27y^3 + 125z^3$

(ii) $64m^3 - 343n^3$

Solution

(i) $27y^3 + 125z^3$
Using identity proved in the above question $x^3 + y^3 = (x + y) (x^2 - xy + y^2)$
$27y^3 + 125z^3 = (3y)^3 + (5z)^3$
= $(3y + 5z) \{(3y)^2 - (3y)(5z) + (5z)^2\}$
=$(3y + 5z) (9y^2 - 15yz + 25z)^2$

(ii) $64m^3 - 343n^3$
Using identity proved in the above question, $x^3 - y^3 = (x - y) (x^2 + xy + y^2)$
$64m^3 - 343n^3= (4m)^3 - (7n)^3$
= $(4m + 7n) \{(4m)^2 + (4m) (7n) + (7n)^2\}$
= $(4m + 7n) (16m^2 + 28mn + 49n)^2$

Question 11

Factorise: $27x^3 + y^3 + z^3 - 9xyz$

Solution

$27x^3 + y^3 + z^3 - 9xyz = (3x)^3 + y^3 + z^3 - 3\times3xyz$
Using identity

$x^3 + y^3 + z^3 - 3xyz = (x + y + z) (x^2 + y^2 + z^2 - xy - yz - xz)$

Therefore,
$27x^3 + y^3 + z^3 - 9xyz$
$= (3x + y + z) \{(3x)^2 + y^2 + z^2 - 3xy - yz - 3xz\}$
$= (3x + y + z) (9x^2 + y^2 + z^2 - 3xy - yz - 3xz)$

Question 12

Verify that: $x^3 + y^3 + z^3 - 3xyz = 1/2(x + y + z) [(x - y)^2 + (y - z)^2 + (z - x)^2]$

Solution

We know that,
$x^3 + y^3 + z^3 - 3xyz = (x + y + z) (x^2 + y^2 + z^2 - xy - yz - xz)$

Multiplying and dividing by 2 on Right hand side
$= 1/2 \times (x + y + z) 2(x^2 + y^2 + z^2 - xy - yz - xz)$
$= 1/2(x + y + z) (2x^2 + 2y^2 + 2z^2 - 2xy - 2yz - 2xz)$
$= 1/2(x + y + z) [(x^2 + y^2 - 2xy) + (y^2 + z^2 - 2yz) + (x^2 + z^2 - 2xz)]$
$= 1/2(x + y + z) [(x - y)^2 + (y - z)^2 + (z - x)^2]$

Question 13

If $x + y + z = 0$, show that $x^3 + y^3 + z^3 = 3xyz$.

Solution

We know that,
$x^3 + y^3 + z^3 - 3xyz = (x + y + z) (x^2 + y^2 + z^2 - xy - yz - xz)$
Now put $(x + y + z) = 0$,
$x^3 + y^3 + z^3 - 3xyz = (0) (x^2 + y^2 + z^2 - xy - yz - xz)$
$x^3 + y^3 + z^3 - 3xyz = 0$

Question 14

Without calculating the cubes, find the value of each of the following:
(i) $(-12)^3 + (7)^3 + (5)^3$
(ii) $(28)^3 + (-15)^3 + (-13)^3$

Answer

From previous question, we know that

If $x + y + z = 0$, then $x^3 + y^3 + z^3 = 3xyz$

(i) $(-12)^3 + (7)^3 + (5)^3$
Let $x = -12$, $y = 7$ and $z = 5$

We observed that, x + y + z = -12 + 7 + 5 = 0

Therefore,

$(-12)^3 + (7)^3 + (5)^3 = 3(-12)(7)(5) = -1260$

(ii)$(28)^3 + (-15)^3 + (-13)^3$
Let x =28, y = -15 and z = -13
We observed that, x + y + z = 28 - 15 - 13 = 0

Therefore.
$(28)^3 + (-15)^3 + (-13)^3 = 3(28)(-15)(-13) = 16380$

Question 15

Give possible expressions for the length and breadth of each of the following rectangles, in which their areas are given:
(i) Area: $25a^2 - 35a + 12$
(ii) Area: $35y^2 + 13y - 12$

Solution

(i) Area: $25a^2 - 35a + 12$

Now Area= Length X Breadth

So, factorizing Area

$25a^2 - 35a + 12$
$= 25a^2 - 15a - 20a + 12$
$= 5a(5a - 3) - 4(5a - 3)$
$= (5a - 4)(5a - 3)$
Possible expression for length = 5a - 4
Possible expression for breadth = 5a - 3

(ii)Area: $35y^2 + 13y - 12$
$35y^2 + 13y - 12$
$= 35y^2 - 15y + 28y - 12$
$= 5y(7y - 3) + 4(7y - 3)$
$= (5y + 4)(7y - 3)$
Possible expression for length = (5y + 4)
Possible expression for breadth = (7y - 3)

Question 16

What are the possible expressions for the dimensions of the cuboids whose volumes are given below?

(i) Volume: $3x^2 - 12x$
(ii) Volume: $12ky^2 + 8ky - 20k$

Answer

(i) Volume: $3x^2 - 12x$

Volume of Cuboids= Length X Breadth X height
Now, $3x^2 - 12x$
$= 3x (x - 4)$
Possible expression for length = 3
Possible expression for breadth = x
Possible expression for height = (x - 4)

(ii) Volume: $12ky^2 + 8ky - 20k$
Volume of Cuboids= Length X Breadth X height
Now,

$12ky^2 + 8ky - 20k$
$= 4k (3y^2 + 2y - 5)$
$= 4k (3y^2 +5y - 3y - 5)$
$= 4k [y (3y +5) - 1(3y + 5)]$
$= 4k (3y +5) (y - 1)$
Possible expression for length = 4k
Possible expression for breadth = (3y +5)
Possible expression for height = (y - 1)