SAMPLE QUESTION PAPER
MATHEMATICS (041)
CLASS XII – 2016-17

Time allowed : **3** hours Maximum Marks : **100**

General Instructions:

(i) **All** questions are compulsory.
(ii) This question paper contains **29** questions.
(iii) Question **1- 4** in **Section A** are very short-answer type questions carrying **1** mark each.
(iv) Question **5-12** in **Section B** are short-answer type questions carrying **2** marks each.
(v) Question **13-23** in **Section C** are long-answer-**I** type questions carrying **4** marks each.
(vi) Question **24-29** in **Section D** are long-answer-**II** type questions carrying **6** marks each.

Section-A

Questions 1 to 4 carry 1 mark each.

1. State the reason why the Relation $R=\left\{(a,b):a\le b^2\right\}$ on the set **R** of real numbers is not reflexive.

2. If A is a square matrix of order 3 and $|2A|=k|A|$, then find the value of k.

3. If $\vec{a}$ and $\vec{b}$ are two nonzero vectors such that $\left|\vec{a}\times\vec{b}\right|=\vec{a}\cdot\vec{b}$, then find the angle between $\vec{a}$ and $\vec{b}$.

4. If $*$ is a binary operation on the set **R** of real numbers defined by $a*b=a+b-2$, then find the identity element for the binary operation $*$.

Section-B

Questions 5 to 12 carry 2 marks each.

5. Simplify $\cot^{-1}\dfrac{1}{\sqrt{x^2-1}}$ for $x<-1$.

6. Prove that the diagonal elements of a skew symmetric matrix are all zeros.

7. If $y=\tan^{-1}\dfrac{5x}{1-6x^2}, -\dfrac{1}{\sqrt{6}}<x<\dfrac{1}{\sqrt{6}}$, then prove that $\dfrac{dy}{dx}=\dfrac{2}{1+4x^2}+\dfrac{3}{1+9x^2}$.

8. If x changes from 4 to 4.01, then find the approximate change in $\log_e x$.

9. Find $\int\left(\dfrac{1-x}{1+x^2}\right)^2 e^x dx$.

10. Obtain the differential equation of the family of circles passing through the points $(a,0)$ and $(-a,0)$.

11. If $\left|\vec{a}+\vec{b}\right| = 60, \left|\vec{a}-\vec{b}\right| = 40$ and $\left|\vec{a}\right| = 22$, then find $\left|\vec{b}\right|$.

12. If $P(A) = \frac{2}{5}, P(B) = \frac{1}{3}, P(A \cap B) = \frac{1}{5}$, then find $P(\bar{A}/\bar{B})$.

Section-C

Questions 13 to 23 carry 4 marks each.

13. If $A = \begin{bmatrix} 1 & -2 \\ 2 & 1 \end{bmatrix}$, then using A^{-1}, solve the following system of equations: $x - 2y = -1, 2x + y = 2$.

14. Discuss the differentiability of the function $f(x) = \begin{cases} 2x-1, x < \frac{1}{2} \\ 3-6x, x \geq \frac{1}{2} \end{cases}$ at $x = \frac{1}{2}$.

OR

For what value of k is the following function continuous at $x = -\frac{\pi}{6}$?

$$f(x) = \begin{cases} \dfrac{\sqrt{3}\sin x + \cos x}{x + \frac{\pi}{6}}, x \neq -\frac{\pi}{6} \\ k, x = -\frac{\pi}{6} \end{cases}$$

15. If $x = a\sin pt, y = b\cos pt$, then show that $\left(a^2 - x^2\right) y \frac{d^2y}{dx^2} + b^2 = 0$.

16. Find the equation of the normal to the curve $2y = x^2$, which passes through the point (2, 1).

OR

Separate the interval $\left[0, \frac{\pi}{2}\right]$ into subintervals in which the function $f(x) = \sin^4 x + \cos^4 x$ is strictly increasing or strictly decreasing.

17. A magazine seller has 500 subscribers and collects annual subscription charges of Rs.300 per subscriber. She proposes to increase the annual subscription charges and it is believed that for every increase of Re 1, one subscriber will discontinue. What increase will bring maximum income to her? Make appropriate assumptions in order to apply derivatives to reach the solution. **Write one important role of magazines in our lives.**

18. Find $\int \frac{\sin x}{\left(\cos^2 x + 1\right)\left(\cos^2 x + 4\right)} dx$.

19. Find the general solution of the differential equation $(1+\tan y)(dx-dy)+2xdy=0$.

OR

Solve the following differential equation: $\left(1+e^{\frac{x}{y}}\right)dx+e^{\frac{x}{y}}\left(1-\frac{x}{y}\right)dy=0$.

20. Prove that $\vec{a}\cdot\{(\vec{b}+\vec{c})\times(\vec{a}+2\vec{b}+3\vec{c})\}=\left[\vec{a}\quad\vec{b}\quad\vec{c}\right]$.

21. Find the values of a so that the following lines are skew: $\frac{x-1}{2}=\frac{y-2}{3}=\frac{z-a}{4},\frac{x-4}{5}=\frac{y-1}{2}=z$.

22. A bag contains 4 green and 6 white balls. Two balls are drawn one by one without replacement. If the second ball drawn is white, what is the probability that the first ball drawn is also white?

23. Two cards are drawn successively with replacement from a well shuffled pack of 52 cards. Find the probability distribution of the number of diamond cards drawn. Also, find the mean and the variance of the distribution.

Section-D

Questions 24 to 29 carry 6 marks each.

24. Let $f:[0,\infty)\to\mathbf{R}$ be a function defined by $f(x)=9x^2+6x-5$. Prove that f is not invertible. Modify, only the codomain of f to make f invertible and then find its inverse.

OR

Let $*$ be a binary operation defined on $Q\times Q$ by $(a,b)*(c,d)=(ac,b+ad)$, where Q is the set of rational numbers. Determine, whether $*$ is commutative and associative. Find the identity element for $*$ and the invertible elements of $Q\times Q$.

25. Using properties of determinants, prove that $\begin{vmatrix}\frac{(a+b)^2}{c} & c & c\\ a & \frac{(b+c)^2}{a} & a\\ b & b & \frac{(c+a)^2}{b}\end{vmatrix}=2(a+b+c)^3$.

OR

If $p \neq 0, q \neq 0$ and $\begin{vmatrix} p & q & p\alpha + q \\ q & r & q\alpha + r \\ p\alpha + q & q\alpha + r & 0 \end{vmatrix} = 0,$ then, using properties of determinants, prove that at least one of the following statements is true: (a) p, q, r are in G. P., (b) α is a root of the equation $px^2 + 2qx + r = 0.$

26. Using integration, find the area of the region bounded by the curves $y = \sqrt{5 - x^2}$ and $y = |x - 1|.$

27. Evaluate the following: $\int_0^{\frac{\pi}{2}} \frac{x \sin x \cos x}{\sin^4 x + \cos^4 x} dx.$

OR

Evaluate $\int_0^4 (x + e^{2x}) dx$ as the limit of a sum.

28. Find the equation of the plane through the point (4, -3, 2) and perpendicular to the line of intersection of the planes x – y + 2z – 3 = 0 and 2x – y -3z = 0. Find the point of intersection of the line $\vec{r} = \hat{i} + 2\hat{j} - \hat{k} + \lambda(\hat{i} + 3\hat{j} - 9\hat{k})$ and the plane obtained above.

29. In a mid-day meal programme, an NGO wants to provide vitamin rich diet to the students of an MCD school .The dietician of the NGO wishes to mix two types of food in such a way that vitamin contents of the mixture contains at least 8 units of vitamin A and 10 units of vitamin C. Food 1 contains 2 units per kg of vitamin A and 1 unit per kg of vitamin C. Food 2 contains 1 unit per Kg of vitamin A and 2 units per kg of vitamin C. It costs Rs 50 per kg to purchase Food 1 and Rs 70 per kg to purchase Food 2. Formulate the problem as LPP and solve it graphically for the minimum cost of such a mixture?

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MATHEMATICS (041)
CLASS XII – 2016-17

Marking Scheme

Section A

1. $\frac{1}{2} > \left(\frac{1}{2}\right)^3 \Rightarrow \left(\frac{1}{2}, \frac{1}{2}\right) \notin R$. Hence, R is not reflexive. [1]

2. $k = 2^3 = 8$ [1]

3. $\sin\theta = \cos\theta \Rightarrow \theta = 45^\circ$ [1]

4. $e \in R$ is the identity element for $*$ if $a * e = e * a = a \forall a \in R \Rightarrow e = 2$ [1]

Section B

5. Let $\sec^{-1} x = \theta$. Then $x = \sec\theta$ and for x < -1, $\frac{\pi}{2} < \theta < \pi$ [1/2]

Given expression = $\cot^{-1}(-\cot\theta)$ [1/2]

= $\cot^{-1}(\cot(\pi - \theta)) = \pi - \sec^{-1} x$ as $0 < \pi - \theta < \frac{\pi}{2}$ [1]

6. Let A be a skew-symmetric matrix. Then by definition $A' = -A$ [1/2]

$\Rightarrow$ *the* (i, j)*th element of* $A' =$ *the* (i, j)*th element of* $(-A)$ [1/2]

$\Rightarrow$ *the* (j, i)*th element of* $A = -$*the* (i, j)*th element of* A [1/2]

For the diagonal elements $i = j \Rightarrow$ *the* (i, i)*th element of* $A = -$*the* (i, i)*th element of* A

$\Rightarrow$ *the* (i, i)*th element of* $A = 0$ Hence, the diagonal elements are all zero. [1/2]

7. $y = \tan^{-1}\frac{3x + 2x}{1 - 3x2x} = \tan^{-1} 3x + \tan^{-1} 2x$ [1]

$\frac{dy}{dx} = \frac{3}{1 + 9x^2} + \frac{2}{1 + 4x^2}$ [1]

8. Let $y = \log_e x, x = 4, \delta x = .01$ [1/2]

$\frac{dy}{dx} = \frac{1}{x}$ [1/2]

$dy = \left(\frac{dy}{dx}\right)_{x=4} \times \delta x = \frac{1}{400} = .0025$ [1]

9. Given integral $= \int \left(\frac{1}{1 + x^2} - \frac{2x}{\left(1 + x^2\right)^2} \right) e^x dx$ [1]

$= \frac{1}{1 + x^2} e^x + c$ as $\frac{d}{dx}\left(\frac{1}{1 + x^2}\right) = \frac{-2x}{\left(1 + x^2\right)^2}$ [1]

10. $x^2+(y-b)^2=a^2+b^2 or, x^2+y^2-2by=a^2$.........(1) [1]

$$2x+2y\frac{dy}{dx}-2b\frac{dy}{dx}=0 \Rightarrow 2b=\frac{2x+2y\frac{dy}{dx}}{\frac{dy}{dx}}\text{.........(2)}$$ [1/2]

Substituting in (1), $(x^2-y^2-a^2)\frac{dy}{dx}-2xy=0$ [1/2]

11. $\left|\vec{a}+\vec{b}\right|^2+\left|\vec{a}-\vec{b}\right|^2=2\left(\left|\vec{a}\right|^2+\left|\vec{b}\right|^2\right)$ [1]

$\Rightarrow\left|\vec{b}\right|^2=2116$ [1/2]

$\Rightarrow\left|\vec{b}\right|=46$ [1/2]

12. $P(\bar{A}/\bar{B})=\frac{P(\bar{A}\cap\bar{B})}{P(\bar{B})}$ [1/2]

$=\frac{1-P(A\cup B)}{1-P(B)}$ [1/2]

$=\frac{1-[P(A)+P(B)-P(A\cap B)]}{1-P(B)}$ [1/2]

$=\frac{7}{10}$ [1/2]

Section C

13. $|A|=5$ [1/2]

$adjA=\begin{bmatrix}1 & 2\\ -2 & 1\end{bmatrix}$ [1+1/2]

$A^{-1}=\frac{adjA}{|A|}=\frac{1}{5}\begin{bmatrix}1 & 2\\ -2 & 1\end{bmatrix}$ [1/2]

Given system of equations is $AX=B,$ where $X=\begin{bmatrix}x\\ y\end{bmatrix}, B=\begin{bmatrix}-1\\ 2\end{bmatrix}$ [1/2]

$X=A^{-1}B=\begin{bmatrix}\frac{3}{5}\\ \frac{4}{5}\end{bmatrix}$ [1/2]

$\Rightarrow x=\frac{3}{5}, y=\frac{4}{5}$ [1/2]

14. $Lf'(\frac{1}{2}) = \lim_{h \to 0^+} \frac{f(\frac{1}{2}-h) - f(\frac{1}{2})}{-h} = \lim_{h \to 0^+} \frac{2(\frac{1}{2}-h)-1-0}{-h} = 2$ [1+1/2]

$Rf'(\frac{1}{2}) = \lim_{h \to 0^+} \frac{f(\frac{1}{2}+h) - f(\frac{1}{2})}{h} = \lim_{h \to 0^+} \frac{3-6(\frac{1}{2}+h)-0}{h} = -6$ [1+1/2]

$Lf'(\frac{1}{2}) \neq Rf'(\frac{1}{2})$, f is not differentiable at $x = \frac{1}{2}$ [1]

OR

$\lim_{x \to -\frac{\pi}{6}} f(x) = \lim_{x \to -\frac{\pi}{6}} \frac{2\sin(x+\frac{\pi}{6})}{x+\frac{\pi}{6}}$ [2]

$= 2$ [1]

$f(-\frac{\pi}{6}) = k$ [1/2]

For the continuity of $f(x)$ at $x = -\frac{\pi}{6}$, $f(-\frac{\pi}{6}) = \lim_{x \to -\frac{\pi}{6}} f(x) \Rightarrow k = 2$ [1/2]

15. $\frac{dx}{dt} = ap\cos pt, \frac{dy}{dt} = -bp\sin pt$ [1]

$\frac{dy}{dx} = \frac{-bp\sin pt}{ap\cos pt} = -\frac{b}{a}\tan pt$ [1/2]

$\frac{d^2y}{dx^2} = \frac{-bp\sec^2 pt}{a}\frac{dt}{dx}$ [1]

$= \frac{-bp\sec^2 pt}{a} \times \frac{1}{pa\cos pt} = \frac{-b^2}{(a^2-x^2)y} \Rightarrow (a^2-x^2)y\frac{d^2y}{dx^2} + b^2 = 0$ [1+1/2]

16. Let the normal be at (x_1, y_1) to the curve $2y = x^2$. $\frac{dy}{dx} = x$ The slope of the normal at

$(x_1, y_1) = \frac{-1}{\left(\frac{dy}{dx}\right)_{(x_1,y_1)}} = \frac{-1}{x_1}$ [1]

The equation of the normal is $y - y_1 = \frac{-1}{x_1}(x - x_1)$ [1/2]

The point (2, 1) satisfies it $1 - y_1 = \frac{-1}{x_1}(2 - x_1) \Rightarrow x_1y_1 = 2$.....(1) [1/2]

Also, $2y_1 = x_1^2$......(2) [1/2]

Solving (1) and (2), we get $x_1 = 2^{\frac{2}{3}}, y_1 = 2^{\frac{1}{3}}$ [1/2]

The required equation of the normal is $x + 2^{\frac{2}{3}}y = 2 + 2^{\frac{2}{3}}$ [1]

OR

$f'(x) = 4\sin^3 x\cos x - 4\cos^3 x\sin x = -\sin 4x$ [1]

$f'(x) = 0 \Rightarrow x = \frac{\pi}{4}$ [1]

In the interval	Sign of f′(x)	Conclusion	Marks
$(0, \frac{\pi}{4})$	-ve as $0 < 4x < \pi$	f is strictly decreasing in $[0, \frac{\pi}{4}]$	[1]
$(\frac{\pi}{4}, \frac{\pi}{2})$	+ve as $\pi < 4x < 2\pi$	f is strictly increasing in $[\frac{\pi}{4}, \frac{\pi}{2}]$	[1]

17. Increase in subscription charges = Rs x, Decrease in the number of subscriber = x. Obviously, x is a whole number. [1/2]

Income is given by y = (500 – x)(300 + x). Let us assume for the time being

$0 \le x < 500, x \in R$ [1]

$\frac{dy}{dx} = 200 - 2x, \frac{dy}{dx} = 0 \Rightarrow x = 100$ [1/2]

$\frac{d^2y}{dx^2} = -2, \left(\frac{d^2y}{dx^2}\right)_{x=100} = -2 < 0$ [1/2]

y is maximum when x = 100, which is a whole number. Therefore, she must increase the subscription charges by Rs 100 to have maximum income. [1/2]

Magazines contribute, a great deal, to the development of our knowledge. Through valuable and subtle critical and commentary articles on culture, social civilization, new life style we learn a lot of interesting things. Through reading magazines, our mind and point of view are consolidated and enriched.

[1]

18. $\cos x = t \Rightarrow -\sin x dx = dt$ The given integral $= -\int \frac{dt}{(t^2+1)(t^2+4)}$ [1]

Put $t^2 = y, \frac{-1}{(y+1)(y+4)} = \frac{A}{y+1} + \frac{B}{y+4}$ [1/2]

$-1 = (y+4)A + B(y+1), 0 = A + B, -1 = 4A + B \therefore A = \frac{-1}{3}, B = \frac{1}{3}$ [1]

The given integral $= -\frac{1}{3}\int \frac{dt}{(t^2+1)} + \frac{1}{3}\int \frac{dt}{(t^2+4)} = -\frac{1}{3}\tan^{-1} t + \frac{1}{6}\tan^{-1}\frac{t}{2} + c$ [1]

$= -\frac{1}{3}\tan^{-1}(\cos x) + \frac{1}{6}\tan^{-1}\frac{\cos x}{2} + c$ [1/2]

19. $\left(1+\tan y\right)dx=(1+\tan y-2x)dy \Rightarrow \dfrac{dx}{dy}+\dfrac{2}{1+\tan y}x=1$ [1]

$I.F.=e^{\int\frac{2dy}{1+\tan y}}=e^{\int\frac{(-\sin y+\cos y)+(\cos y+\sin y)}{\cos y+\sin y}dy}=e^{\log_e(\cos y+\sin y)+y}=(\cos y+\sin y)e^y$ [2]

$x(\cos y+\sin y)e^y=\int(\cos y+\sin y)e^y dy \Rightarrow x(\cos y+\sin y)e^y=e^y\sin y+c$ [1]

OR

We have $(1+e^{\frac{x}{y}})dx=\left(\dfrac{x}{y}-1\right)e^{\frac{x}{y}}dy \Rightarrow \dfrac{dx}{dy}=\dfrac{\left(\dfrac{x}{y}-1\right)e^{\frac{x}{y}}}{(1+e^{\frac{x}{y}})}=f(\dfrac{x}{y})$, hence homogeneous [1/2]

$x=vy, \dfrac{dx}{dy}=v+y\dfrac{dv}{dy}$ [1/2]

$\int\dfrac{1+e^v}{e^v+v}dv=-\int\dfrac{dy}{y}$ [1]

$\log_e\left|e^v+v\right|=-\log_e|y|+\log_e c$ [1]

$\Rightarrow \log_e\left|(e^v+v)y\right|=\log_e c$ [1/2]

$\Rightarrow (e^v+v)y=\pm c=A \Rightarrow (e^{\frac{x}{y}}+\dfrac{x}{y})y=A$, the general solution [1/2]

20. LHS $=\vec{a}\cdot(\vec{b}\times\vec{a}+2\vec{b}\times\vec{b}+3\vec{b}\times\vec{c}+\vec{c}\times\vec{a}+2\vec{c}\times\vec{b}+3\vec{c}\times\vec{c})$ [1]

$=\vec{a}\cdot(\vec{b}\times\vec{a})+3\vec{a}\cdot(\vec{b}\times\vec{c})+\vec{a}\cdot(\vec{c}\times\vec{a})+2\vec{a}\cdot(\vec{c}\times\vec{b})$ as $\vec{b}\times\vec{b}=\vec{c}\times\vec{c}=\vec{0}$ [1]

$=3\left[\vec{a}\quad\vec{b}\quad\vec{c}\right]+2\left[\vec{a}\quad\vec{c}\quad\vec{b}\right]$ [1]

$=3\left[\vec{a}\quad\vec{b}\quad\vec{c}\right]-2\left[\vec{a}\quad\vec{b}\quad\vec{c}\right]=\left[\vec{a}\quad\vec{b}\quad\vec{c}\right]$ [1]

21. As $2:3:4\neq 5:2:1$, the lines are not parallel [1/2]

Any point on the first line is $(2\lambda+1, 3\lambda+2, 4\lambda+a)$ [1]

Any point on the second line is $(5\mu+4, 2\mu+1, \mu)$ [1]

Lines will be skew, if, apart from being non parallel, they do not intersect. There must not exist a pair of values of λ, μ, which satisfy the three equations simultaneously:

$2\lambda+1=5\mu+4, 3\lambda+2=2\mu+1, 4\lambda+a=\mu$Solving the first two equations, we get $\lambda=-1, \mu=-1$

[1]

These values will not satisfy the third equation if $a\neq 3$ [1/2]

22. Let $E_1=$First ball drawn is white, $E_2=$First ball drawn is green, $A=$Second ball drawn is white [1]

The required probability, by Bayes' Theorem, =

$$P(E_1/A)=\frac{P(E_1)\times P(A/E_1)}{P(E_1)\times P(A/E_1)+P(E_2)\times P(A/E_2)}$$ [1]

$$=\frac{\frac{6}{10}\times\frac{5}{9}}{\frac{6}{10}\times\frac{5}{9}+\frac{4}{10}\times\frac{6}{9}}=\frac{5}{9}$$ [2]

23. Let X denote the random variable. X=0, 1, 2 n = 2, p = ¼, q = ¾ [1/2]

x_i	0	1	2	Total	Marks
p_i	${}^2C_0\left(\frac{3}{4}\right)^2=\frac{9}{16}$	${}^2C_1\frac{1}{4}\left(\frac{3}{4}\right)=\frac{6}{16}$	${}^2C_2\left(\frac{1}{4}\right)^2=\frac{1}{16}$		[1+1/2]
x_ip_i	0	6/16	2/16	1/2	
$x_i^2p_i$	0	6/16	4/16	5/8	[1/2]

Mean = $\sum x_ip_i=\frac{1}{2}$ [1/2]

Variance = $\sum x_i^2p_i-\left(\sum x_ip_i\right)^2$ [1/2]

$=\frac{5}{8}-\frac{1}{4}=\frac{3}{8}$ [1/2]

Section D

24. $\forall x\in[0,\infty),\ y=9x^2+6x-5=(3x+1)^2-6\geq-5$..........(1) Range f = $[-5,\infty)\neq$ codomain f, hence, f is not onto and hence, not invertible [2]

Let us take the modified codomain f = $[-5,\infty)$ [1/2]

Let us now check whether f is one-one. Let $x_1,x_2\in[0,\infty)$ such that

$f(x_1)=f(x_2)\Rightarrow(3x_1+1)^2-6=(3x_2+1)^2-6\Rightarrow3x_1+1=3x_2+1\Rightarrow x_1=x_2$ Hence, f is one-one.

Since, with the modified codomain = the Range f, f is both one-one and onto, hence invertible.

[1+1/2]

From (1) above, for any y $\in[-5,\infty),(3x+1)^2=y+6\Rightarrow x=\frac{\sqrt{y+6}-1}{3}$ [1]

$f^{-1}:[-5,\infty)\to[0,\infty),\ f^{-1}(y)=\frac{\sqrt{y+6}-1}{3}$ [1]

OR

Let $(a,b),(c,d)\in Q\times Q$. Then b + ad may not be equal to d + cb. We find that

$(1,2)*(2,3)=(2,5),(2,3)*(1,2)=(2,7)\neq(2,5)$ Hence, $*$ is not commutative. [1]

Let

$(a,b),(c,d),(e,f)\in Q\times Q,((a,b)*(c,d))*(e,f)=(ace,b+ad+acf)=(a,b)*((c,d))*(e,f))$

Hence, $*$ is associative. [1]

$(x,y)\in Q\times Q$ is the identity element for $*$ if

$(x,y)*(a,b)=(a,b)*(x,y)=(a,b)\forall(a,b)\in Q\times Q$ i.e.,

$(xa,y+xb)=(ax,b+ay)=(a,b) i.e., xa=a, y+xb=b+ay=b$, (x, y) = (1, 0) satisfies these equations. Hence, (1, 0) is the identity element for $*$ [2]

$(c,d)\in Q\times Q$ is the inverse of $(a,b)\in Q\times Q$ if

$(c,d)*(a,b)=(a,b)*(c,d)=(1,0), i.e., (ac,b+ad)=(ca,d+cb)=(1,0)\Rightarrow c=\frac{1}{a}, d=\frac{-b}{a}$. The inverse of $(a,b)\in Q\times Q, a\neq 0$ is $(\frac{1}{a},\frac{-b}{a})$ [2]

25. LHS = $\frac{1}{abc}\begin{vmatrix}(a+b)^2 & c^2 & c^2\\ a^2 & (b+c)^2 & a^2\\ b^2 & b^2 & (c+a)^2\end{vmatrix}$ [1/2]

$$=\frac{1}{abc}\begin{vmatrix}(a+b+c)(a+b-c) & 0 & c^2\\ 0 & (b+c+a)(b+c-a) & a^2\\ (b+c+a)(b-c-a) & (b+c+a)(b-c-a) & (c+a)^2\end{vmatrix}(C_1\to C_1-C_3, C_2\to C_2-C_3)$$ [1]

$$=\frac{(a+b+c)^2}{abc}\begin{vmatrix}(a+b-c) & 0 & c^2\\ 0 & (b+c-a) & a^2\\ (b-c-a) & (b-c-a) & (c+a)^2\end{vmatrix}$$ [1/2]

$$=\frac{(a+b+c)^2}{abc}\begin{vmatrix}(a+b-c) & 0 & c^2\\ 0 & (b+c-a) & a^2\\ (-2a) & (-2c) & 2ca\end{vmatrix}(R_3\to R_3-(R_1+R_2))$$ [1]

$$=\frac{(a+b+c)^2}{abcca}\begin{vmatrix}(ac+bc-c^2) & 0 & c^2\\ 0 & (ba+ca-a^2) & a^2\\ (-2ac) & (-2ca) & 2ca\end{vmatrix}$$ [1/2]

$$=\frac{(a+b+c)^2}{abcca}\begin{vmatrix}(ac+bc) & c^2 & c^2\\ a^2 & (ba+ca) & a^2\\ 0 & 0 & 2ca\end{vmatrix}(C_1\to C_1+C_3, C_2\to C_2+C_3)$$ [1]

$$= \frac{(a+b+c)^2 2c^2a^2}{abcca} \begin{vmatrix} (a+b) & c & c \\ a & (b+c) & a \\ 0 & 0 & 1 \end{vmatrix} (C_1 \to C_1 + C_3, C_2 \to C_2 + C_3)$$ [1]

$$= \frac{(a+b+c)^2 2}{b}\left(ab+ac+b^2+bc-ac\right) = 2(a+b+c)^3$$ [1/2]

OR

Given equation $\Rightarrow \frac{1}{pq} \begin{vmatrix} pq & q^2 & pq\alpha + q^2 \\ pq & pr & pq\alpha + pr \\ p\alpha + q & q\alpha + r & 0 \end{vmatrix} = 0$ [1]

$$\Rightarrow \frac{1}{pq} \begin{vmatrix} 0 & q^2 - pr & q^2 - pr \\ pq & pr & pq\alpha + pr \\ p\alpha + q & q\alpha + r & 0 \end{vmatrix} = 0 \ (R_1 \to R_1 - R_2)$$ [1]

$$\Rightarrow \frac{q^2 - pr}{pq} \begin{vmatrix} 0 & 1 & 1 \\ pq & pr & pq\alpha + pr \\ p\alpha + q & q\alpha + r & 0 \end{vmatrix} = 0$$ [1]

$$\Rightarrow \frac{q^2 - pr}{pq} p \begin{vmatrix} 0 & 1 & 1 \\ q & r & q\alpha + r \\ p\alpha + q & q\alpha + r & 0 \end{vmatrix} = 0$$ [1]

$$\Rightarrow \frac{q^2 - pr}{q} \begin{vmatrix} 0 & 0 & 1 \\ q & -q\alpha & q\alpha + r \\ p\alpha + q & q\alpha + r & 0 \end{vmatrix} = 0 (C_2 \to C_2 - C_3)$$ [1]

$\Rightarrow \frac{q^2 - pr}{q}(q^2\alpha + rq + pq\alpha^2 + q^2\alpha) = 0 \Rightarrow (q^2 - pr)(2q\alpha + r + p\alpha^2) = 0 \Rightarrow q^2 - pr = 0$ (i.e., p, q, r are in GP) or $2q\alpha + r + p\alpha^2 = 0$(i.e., α is a root of the equation $2qx + r + px^2 = 0$ [1]

26.

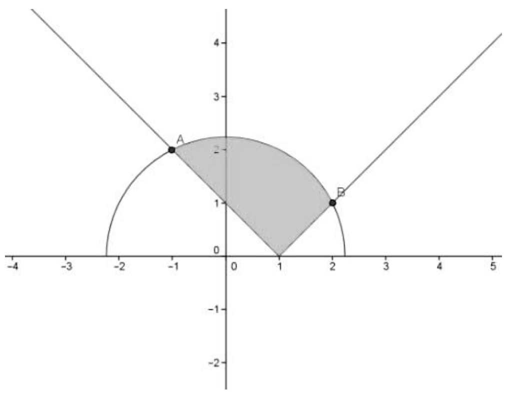


Figure [1 Marks]

Solving $y = \sqrt{5 - x^2}, y = |x-1|$ we get $(x-1)^2 = 5 - x^2 \Rightarrow x^2 - x - 2 = 0 \Rightarrow x = 2, -1$ [1]

The required area = the shaded area = $\int_{-1}^{1}(\sqrt{5-x^2}-(1-x))dx+\int_{1}^{2}(\sqrt{5-x^2}-(x-1))dx$ [2]

$$=\int_{-1}^{2}(\sqrt{5-x^2}dx-\int_{-1}^{1}(1-x)dx+\int_{1}^{2}(-(x-1))dx=\frac{1}{2}\left[x\sqrt{5-x^2}+5\sin^{-1}\frac{x}{\sqrt{5}}\right]_{-1}^{2}-\left[x-\frac{x^2}{2}\right]_{-1}^{1}-\left[\frac{x^2}{2}-x\right]_{1}^{2}$$

[1+ ½]

$$=\left[-\frac{1}{2}+\frac{5}{2}(\sin^{-1}\frac{2}{\sqrt{5}}+\sin^{-1}\frac{2}{\sqrt{5}})\right]\text{sq units}$$ [1/2]

27. $I=\int_{0}^{\frac{\pi}{2}}\frac{x\sin x\cos x}{\sin^4 x+\cos^4 x}dx=\int_{0}^{\frac{\pi}{2}}\frac{(\frac{\pi}{2}-x)\sin(\frac{\pi}{2}-x)\cos(\frac{\pi}{2}-x)}{\sin^4(\frac{\pi}{2}-x)+\cos^4(\frac{\pi}{2}-x)}dx,\ I=\int_{0}^{\frac{\pi}{2}}\frac{(\frac{\pi}{2}-x)\cos x\sin x}{\cos^4 x+\sin^4 x}dx$ [1]

$$2I=\int_{0}^{\frac{\pi}{2}}\frac{(\frac{\pi}{2})\cos x\sin x}{\cos^4 x+\sin^4 x}dx$$ [1/2],

$$2I=(\frac{\pi}{2})[\int_{0}^{\frac{\pi}{4}}\frac{\cos x\sin x}{\cos^4 x+\sin^4 x}dx+\int_{\frac{\pi}{4}}^{\frac{\pi}{2}}\frac{\cos x\sin x}{\cos^4 x+\sin^4 x}dx]=(\frac{\pi}{2})[\int_{0}^{\frac{\pi}{4}}\frac{\tan x\sec^2 x}{1+\tan^4 x}dx+\int_{\frac{\pi}{4}}^{\frac{\pi}{2}}\frac{\cos ec^2 x\cot x}{\cot^4 x+1}dx]$$ [2]

$$2I=(\frac{\pi}{4})[\int_{0}^{1}\frac{1}{1+t^2}dt-\int_{1}^{0}\frac{1}{1+p^2}dp]\text{substituting}$$

$\tan^2 x=t,\cot^2 x=p\Rightarrow 2\tan x\sec^2 xdx=dt,-2\cot x\cos ec^2 xdx=dp$ [1]

$$2I=(\frac{\pi}{4})[\tan^{-1}t]_0^1+(\frac{\pi}{4})[\tan^{-1}p]_0^1=\frac{\pi^2}{8}$$ [1]

$$I=\frac{\pi^2}{16}$$ [1/2]

OR

$f(x)=x+e^{2x},\int_{0}^{4}f(x)dx=\lim_{n\to\infty,h\to 0}h\sum_{r=1}^{n}f(rh),nh=4$ [1]

$f(rh)=rh+e^{2rh},\sum_{r=1}^{n}f(rh)=h\sum_{1}^{n}r+\sum_{1}^{n}e^{2rh}$ [1]

$$=h\frac{n(n+1)}{2}+e^{2h}\frac{e^{2nh}-1}{e^{2h}-1}$$ [2]

$$\int_{0}^{4}f(x)dx=\lim_{n\to\infty,h\to 0}[nh\frac{nh+h}{2}+e^{2h}\frac{e^8-1}{\frac{e^{2h}-1}{2h}}\times\frac{1}{2}]$$ [1]

$$=\lim_{h\to 0}[4\frac{4+h}{2}+e^{2h}\frac{e^8-1}{\frac{e^{2h}-1}{2h}}\times\frac{1}{2}]=8+\frac{e^8-1}{2}$$ [1]

28. $\vec{n} = \vec{b}_1 \times \vec{b}_2 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -1 & 2 \\ 2 & -1 & -3 \end{vmatrix} = 5\hat{i} + 7\hat{j} + \hat{k}$ [2]

The equation of the plane is $\vec{r} \cdot \vec{n} = (5\hat{i} + 7\hat{j} + \hat{k}) \cdot (4\hat{i} - 3\hat{j} + 2\hat{k}), i.e., \vec{r} \cdot (5\hat{i} + 7\hat{j} + \hat{k}) = 1$ [1]

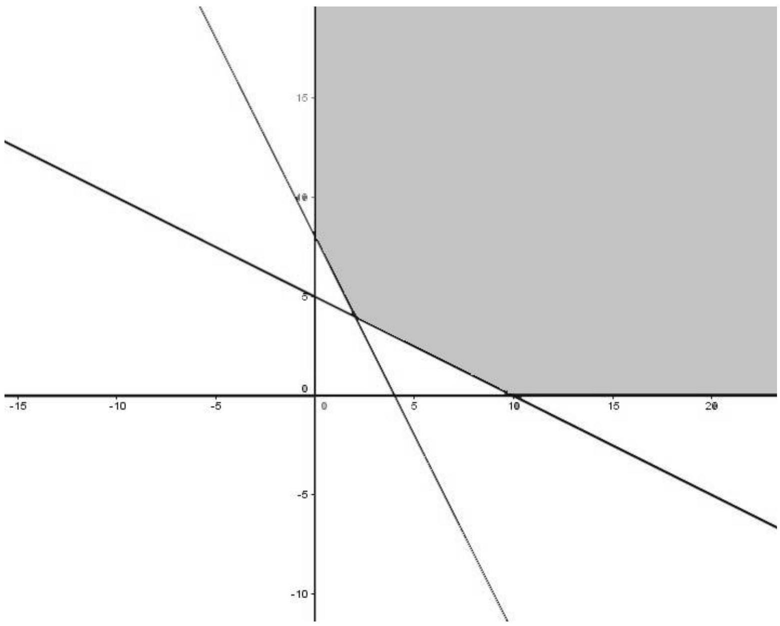
The position vector of any point on the given line is $(1+\lambda)\hat{i} + (2+3\lambda)\hat{j} + (-1-9\lambda)\hat{k}$ [1]

We have $(1+\lambda)5 + (2+3\lambda)7 + (-1-9\lambda)1 = 1$ [1]

$\lambda = -1$ [1/2]

The position vector of the required point is $-\hat{j} + 8\hat{k}$ [1/2]

29. Let x kg of Food 1 be mixed with y kg of Food 2. Then to minimize the cost, C = 50x + 70y subject to the following constraints: $2x + y \geq 8, x + 2y \geq 10, x \geq 0, y \geq 0$ [2]



Graph [2]

At	C	Marks
(0, 8)	Rs 560	[1]
(2,4)	Rs 380	
(10.0)	Rs 500	

In the half plane 50x + 70y < 380, there is no point common with the feasible region. Hence, the minimum cost is Rs 380. [1]

--0-0-0--