Complex Analysis Part 1

Complex Variables

- A function is said to be analytic in a domain D if it is single valued and differentiable at every point in the domain D.
- Points in a domain at which function is not differentiable are singularities of the function in domain D.
- Cauchy Riemann conditions for a function *f(z)=u(x,y)+iv(x,y)* to be analytic at point z
  $\frac{\partial u}{\partial x} = \frac{\partial v}{\partial y}$
  $\frac{\partial v}{\partial x} = -\frac{\partial u}{\partial y}$
- Cauchy Riemann equations in polar form are
  $\frac{\partial u}{\partial r} = \frac{1}{r}\frac{\partial v}{\partial \theta}$
  $\frac{1}{r}\frac{\partial u}{\partial \theta} = -\frac{\partial v}{\partial r}$

Cauchy' Theorem If *f(z)* is an analytic function of z and *f'(z)* is continuous at each point within and on a closed contour C then
$\oint_C f(z)dz = 0$

Green's Theorem
If *M(x,y)* and *N(x,y)* are two functions of x and y and have continous derivatives
$\oint_C (Mdx + Ndy) = \iint_S \left(\frac{\partial N}{\partial x} - \frac{\partial M}{\partial y}\right)\delta x \delta y$

Theorem:-
If function *f(z)* is not analytic in the whole region enclosed by a closed contour C but it is analytic in the region bounded between two contours C_1 and C_2 then
$\int_C f(z)dz = \int_{C_1} f(z)dz + \int_{C_2} f(z)dz$

Cauchy's Integral Formula

If *f(z)* is an analytic function on and within the closed contour C the value of *f(z)* at any point z=a inside C is given by the following contour integral
$f(a) = \frac{1}{2\pi i}\oint_C \frac{f(z)}{z-a}dz$

Cauchy's Integral Formula for derivative of an analytic function

If *f(z)* is an analytic function in a region R , then its derivative at any point z=a is given by
$f'(a) = \frac{1}{2\pi i}\oint_C \frac{f(z)}{(z-a)^2}dz$
generalizing it we get
$f^n(a) = \frac{n!}{2\pi i}\oint_C \frac{f(z)}{(z-a)^{n+1}}dz$

Morera Theorem

It is inverse of Cauchy's theorem. If *f(z)* is continuous in a region R and if $\oint f(z)dz$ taken around a simple closed contour in region R is zero then *f(z)* is an analytic function.

Cauchy's inequality

If *f(z)* is an analytic function within a circle C i.e., $|z-a| = R$ and if $|f(z)| \leq M$ then
$|f^n(a)| \leq \frac{Mn!}{R^n}$