Complex Analysis part 2

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Liouville's Theorem

If a function $f(z)$ is analytic for all finite values of z, and is bounded then it is a constant. Note:- $e^{z+2\pi i} = e^z$

Taylor's Theorem

If a function $f(z)$ is analytic at all points inside a circle C, with its centre at point a and radius R then at each point z inside C

$f(z) = f(a) + (z-a)f'(a) + \frac{1}{2!}(z-a)^2 f''(a) + ... + \frac{1}{n!}(z-a)^n f^n(a)$

Taylor's theorem is applicable when function is analytic at all points inside a circle.

Laurent Series

If $f(z)$ is analytic on C_1 and C_2 and in the annular region R bounded by the two concentric circles C_1 and C_2 of radii r_1 and r_2 $(r_1 > r_2)$ with their centre at a then for all z inside R

$f(z) = a_0 + a_1(z-a) + a_2(z-a)^2 + ... + \frac{b_1}{(z-a)} + \frac{b_2}{(z-a)^2} + ...$ where,

$a_n = \frac{1}{2\pi i}\int_{C_1} \frac{f(w)dw}{(w-a)^{(n+1)}}$ $b_n = \frac{1}{2\pi i}\int_{C_1} \frac{f(w)dw}{(w-a)^{(-n+1)}}$

Singular points

If a function $f(z)$ is not analytic at point z=a then z=a is known as a singular point or there is a singularity of $f(z)$ at z=a for example

$f(z) = \frac{1}{z-2}$ z=2 is a singularity of $f(z)$

Pole of order m

If $f(z)$ has singularity at z=a then from laurent series expansion

$$f(z) = a_0+a_1(z-a)+a_2(z-a)^2+.........+\frac{b_1}{(z-a)}+\frac{b_2}{(z-a)^2}+.....+\frac{b_m}{(z-a)^m}+\frac{b_{m+1}}{(z-a)^{m+1}}$$

if $b_{m+1} = b_{m+2} = 0$ then

$$f(z) = a_0 + a_1(z-a) + a_2(z-a)^2 + ... + \frac{b_1}{(z-a)} + \frac{b_2}{(z-a)^2} + ... + \frac{b_m}{(z-a)^m}$$

and we say that function $f(z)$ is having a pole of order m at z=a. If m=1 then point z=a is a simple pole.

Residue

The constant b_1 , the coefficent of $(z-z_0)^{-1}$, in the Laurent series expansion is called the residue of $f(z)$ at singularity $z = z_0$

$b_1 = Res_{z=z_0} f(z) = \frac{1}{2\pi i} \int_{C_1} f(z)dz$

Methods of finding residues

- Residue at a simple pole

  if $f(z)$ has a simple pole at z=a then $Resf(a) = \lim_{z\to a}(z-a)f(z)$

- If $f(z) = \frac{\Phi(z)}{\Psi(z)}$ and $\Psi(a) = 0$ then $Resf(a) = \frac{\Phi(z)}{\Psi'(z)}$

- Residue at pole of order m

  If $f(z)$ is a pole of order m at z=a then

  $Resf(a) = \frac{1}{(m-1)!}\left\{\frac{d^{m-1}}{dz^{m-1}}(z-a)^m f(z)\right\}_{z=a}$

Residue Theorem

If $f(z)$ is analytic in closed contour C excapt at finite number of points (poles) within C, then

$\int_C f(z)dz = 2\pi i$[*sum of the residues at poles within C*]

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[1]This documeny is created by http://physicscatalyst.com