Electric Current, Resistance and Resistivity

In This small e-book we will learn all we need to know about current electricity but in short and then we'll have some questions discussed along with their solutions. If you want detailed notes on the subject you can always visit our website http://physicscatalyst.com . So you can use it as a revision notes on current electricity before your exams.

Brief Notes

- Electric current is basically defined as the state of motion of electric charge and Quantitatively electric current is defined as the time rate of flow of the net charge of the area of cross-section of the conductor i.e.,
  Electric current = Total charge flowing / time taken
- SI unit of the current is Ampere (A). 1 Ampere= 1 Coulomb/ 1 sec=1 Cs^{-1}.
- Current through any conductor is said to be 1 ampere, if 1 C of charge is flowing through the conductor in 1 sec.
- Direction of electric current is in the direction of the flow of positive charged carriers and this current is known as **conventional current**.
- Electric current is a scalar quantity.
- The **current density** at a point in the conductor is defined as the current per unit cross-section area. Thus if the charge is flowing per unit time uniformly over the area of cross-section A of the conductor, then current density J at any point on that area is defined as
  $$J = \frac{I}{A}$$
- Current density is a vector quantity unlike electric current.
- Unit of current density is Ampere/meter^2 (Am^{-2})
- Small velocity imposed on the random motion of electrons in a conductor on the application of electric field is known as **drift velocity** and is given by
  $$v_d = \frac{eE\tau}{m}$$
  Where τ is the average time interval for which electron accelerates, E is the applied electric field, e is the charge on the electron and m is the mass of the electron.
- This drift velocity is also defined as the velocity with which free electrons gets drifted towards the positive end of the conductor under the influence of externally applied electric field.

- Relation between drift velocity and electric current is
  $I = neAv_d$
  And in terms of current density
  $j = nev_d$
  Where, n are the number of free electrons per unit volume moving with the drift velocity v_d.
- **Statement of Ohm's Law**
  'if the physical state of the conductor (Temperature and mechanical strain etc) remains unchanged ,then current flowing through a conductor is always directly proportional to the potential difference across the two ends of the conductor. Mathematically
  $V \propto I$ or $V=IR$
  Where constant of proportionality R is called the electric resistance or simply resistance of the conductor
- Electric resistance of a conductor is the obstruction offered by the conductor to the flow of the current through it.
- SI unit of resistance is ohm (Ω) where 1 Ohm=1 volt/1 Ampere or $1\Omega=1VA^{-1}$ and dimension of resistance is $[ML^2T^{-3}A^{-2}]$.
- Electric resistivity is defined as the ratio of electric field intensity at any point in the conductor and the current density at that point i.e.,
  $\rho = \frac{E}{j} = \frac{RA}{l} = \frac{m}{ne^2\tau}$
  The greater the resistivity of the material, greater would be the field needed to establish a given current density.
- Perfect conductor have zero resistivity and for perfect insulators resistivity would be infinite
- The reciprocal of resistivity is called conductivity and is represented by σ.
- σ is defined as $\sigma=1/\rho$. Since $\rho=E/J$ or $\sigma=J/E$ so, $J=\sigma E$
- Temperature variation of the resistance can be given as
  $R=R(T_0)[1 + \alpha(T-T_0)]$
- For n numbers of resistors connected in series equivalent resistance would be
  $R_{eq}=R_1+R_2+R_3+..........................+R_n$
- Value of resistance of the series combination is always greater than the value of largest individual radiances.

- If there are n number of resistances connected in parallel combination, then equivalent resistance would be reciprocal of
$$\frac{1}{R_{eq}}=\frac{1}{R_1}+\frac{1}{R_2}+\frac{1}{R_3}+\ldots\ldots\ldots\ldots\ldots+\frac{1}{R_n}$$
- Value of equivalent resistances for capacitors connected in parallel combination is always less than the value of the smallest resistance in circuit.

How to simplify circuits with resistors

1. In any given circuit first of all recognize the resistances connected in series then by summing the individual resistances draw a new, simplified circuit diagram.

2. For series combination of resistances equivalent resistance is given by the equation
$R_{eq}= R_1 + R_2+R_3$
the current in each resistor is the same when connected in parallel combination.

3. Then recognize the resistances connected in parallel and find the equivalent resistances of parallel combinations by summing the reciprocals of the resistances and then taking the reciprocal of the result. Draw the new, simplified circuit diagram.
$(1/R)=(1/R_1)+(1/R_2)+(1/R_2)$

4. Remember that for resistors connected in parallel combination 'The potential difference across each resistor is the same'.

5. Repeat the first two steps as required, until no further combinations can be made using resistances. If there is only a single battery in the circuit, this will usually result in a single equivalent resistor in series with the battery.

6. Use Ohm's Law, V= IR, to determine the current in the equivalent resistor. Then work backwards through the diagrams, applying the useful facts listed in step 1 or step 2 to find the currents in the other resistors. (In more complex circuits, Kirchhoff's rules can be applied).

Questions And Answers

Question 1

Two wires a and b , each of length 40 m and area of cross section $10^{-7}m^2$, are joined in series and potential difference of 60 Volt is applied between the ends of combined wire. Their resistances are respectively 40 and 20 ohm. Determine for each wire

(a) Specific resistance
(b) Electric field
(c) Current density

Solution 1

(a) The specific resistance of the material of the wire is given by

$$\rho = \frac{RA}{l}$$

Where, R is the resistance, l is the length and A is the area of cross-section of the wire. Substituting the values we get,

$$\rho_a = 40\Omega \times \frac{10^{-7}m^2}{40m} = 1.0 \times 10^{-7}\Omega - m$$

And

$$\rho_b = 20\Omega \times \frac{10^{-7}m^2}{40m} = 5.0 \times 10^{-8}\Omega - m$$

(b) Total resistance $R=R_a+R_b=40+20=60$ ohm. The current in the wires is $i=V/R = 60V/60$ ohm $= 1$ A.

Therefore potential difference between ends of wires a and b are respectively

$V_a= i\times R_a=1.0\ A\times 40\Omega = 40V$

$V_b= i\times R_b=1.0\ A\times 20\Omega =20V$

Electric fields in these wires are

$$E_a = \frac{V_a}{l_a} = \frac{40\ V}{40\ m} = 1.0\ V/m$$

And ,

$$E_b = \frac{V_b}{l_b} = \frac{20\ V}{40\ m} = 0.5\ V/m$$

(c) The current in each wire is the same. Also, the area of cross-section of each wire is same. Hence the current density in each wire is

$$j_a = j_b = \frac{i}{A} = 1.0 \times 10^7 Am^2$$

Question 2
A cylindrical wire is stretched to increase its length by 10%. Calculate the percentage increase in the resistance of this wire.
Solution 2
Suppose the initial length of the wire is l. The increase in length is$l \times \frac{10}{100} = 0.1l$.
Therefore, the length of the wire after stretching is
$l^{'} = l + 0.1l = 1.1l$
Or,
$\frac{l'}{l} = 1.1$ ... (1)

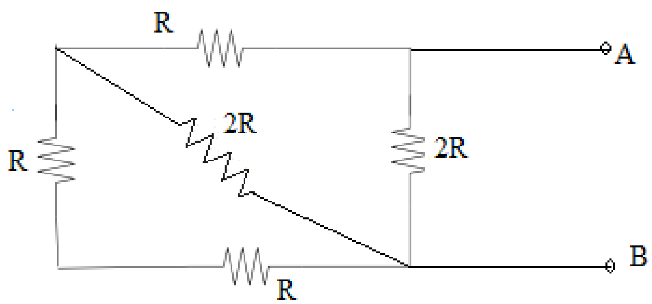
Let the initial cross-section area of the wire be A, and after stretching A'. The volume of the wire will remain unchanged. That is
$Al = A'l'$
Or,
$\frac{A}{A'} = \frac{l'}{l}$ ... (2)
If ρ is the specific resistance of the material of the wire, then the resistance of the wire before stretching is
$R = \rho\frac{l}{A}$
And that after stretching is
$R^{'} = \rho\frac{l'}{A'}$
But from equation (2) A/A'=l'/l
Therefore,
$\frac{R'}{R} = \left(\frac{l'}{l}\right)^2$
Substituting the value of l'/l from equation 1 we get
$\frac{R'}{R} = (1.1)^2 = 1.21$
Percentage increase in resistance is
$\frac{(R^{'} - R)}{R} \times 100 = \left(\frac{R'}{R} - 1\right) \times 100 = (1.21 - 1) \times 100 = 21\%$

Question 3
Two 120V light bulbs, one of 25 W and other of 200 W were connected in series across a 240 V line. One bulb burns out almost instantly, Which one was burnt and why?
Solution 3
As

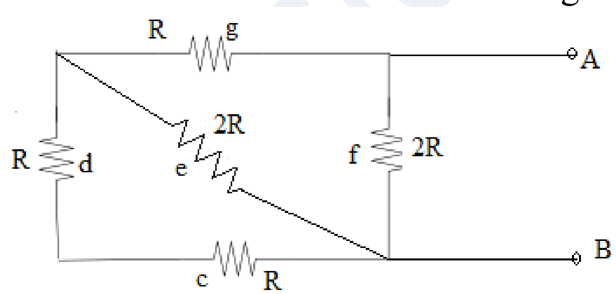
$$P = \frac{V^2}{R}$$

25 W bulb is having more resistance. In series combination same current flows through both the bulbs. So the 25 W bulb will develop more heat and burns out instantaneously.

Question 4
Find the equivalent resistance between point A and B in following combination of resistors



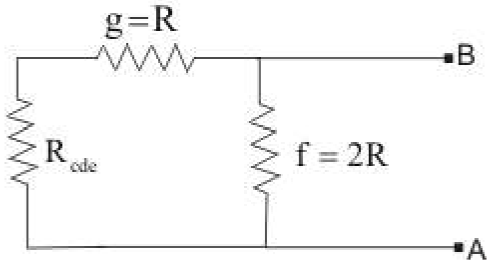
Solution 4
Let us first label the resistors in the figure



Resistors c and d are connected in series combination
Thus R_{cd}=R+R=2R
Now R_{cd} is connected in parallel with resistors e
So equivalent resistance

$$\frac{1}{R_{cde}} = \frac{1}{2R} + \frac{1}{2R}$$

Or
R_{cde}=R
Now the above figure is reduced to below figure



R_{cde} and g are in series
So R_{cdeg}=2R
Now R_{cdeg} and F in parallel
=>

$$\frac{1}{R_{AB}} = \frac{1}{2R} + \frac{1}{2R}$$

R_{ab}=RΩ

Question 5
α_1 and α_2 are the temperature coefficient of the two resistors R_1 and R_2 at any temperature $T_0{}^0$ C. Find the equivalent temperature coefficient of their equivalent resistance if both R_1 and R_2 are connected in series combination. Assume that α_1 and α_2 remain same with change in temperature.
Solution 5
At t= $T_0{}^0$ C, equivalent resistance of resistors connected in series is
R_{eq}= R_1 + R_2

At t= T^0 C
R'_{eq}= R'_1 + R'_2

$$= R_1[1+\alpha_1(T-T_0)] + R_2[1+\alpha_2(T-T_0)]$$

$$= R_1 + R_2 + (R_1\alpha_1 + R_2\alpha_2)(T-T_0) \quad \text{---(1)}$$

If α_{eq} is equivalent temperature coefficient in series combination then

$$R'_{eq} = R_{eq}[1+\alpha_{eq}(T-T_0)]$$

$$= (R_1 + R_2)[1+\alpha_{eq}(T-T_0)] \quad \text{---(2)}$$

From equation 1 and 2,we get

$$R_1 + R_2 + (R_1\alpha_1 + R_2\alpha_2)(T-T) = (R_1 + R_2)[1+\alpha_{eq}(T-T_0)]$$

Solving the equation we get

$$\alpha_{eq} = \frac{R_1\alpha_1 + R_2\alpha_2}{R_1 + R_2}$$

Again find the α_{eq} for the parallel combination of resistors, Answer shall be

$$\alpha_{eq} = \frac{R_1\alpha_2 + R_2\alpha_1}{R_1 + R_2}$$