

Quadratic Equations

Question 1

Check whether the following are quadratic equations

- i) $(x+1)^2=2(x-3)$
- ii) $x^2-2x=(-2)(3-x)$
- iii) $(x-2)(x+1)=(x-1)(x+3)$
- iv) $(x-3)(2x+1)=x(x+5)$
- v) $(2x-1)(x-3)=(x+5)(x-1)$
- vi) $x^2+3x+1=(x-2)^2$
- vii) $(x+2)^3=2x(x^2-1)$
- viii) $x^3-4x^2-x+1=(x-2)^3$

Question 2

Represent the following situations in the form of quadratic equations :

- i) The area of a rectangular plot is 528 m^2 . The length of the plot (in meters) is one more than twice its breadth. We need to find the length and breadth of the plot.
- ii) The product of two consecutive positive integers is 306. We need to find the Integers.
- iii) Rohan's mother is 26 years older than him. The product of their ages (in years) 3 years from now will be 360. We would like to find Rohan's present age.
- iv) A train travels a distance of 480 km at a uniform speed. If the speed had been 8 km/h less, then it would have taken 3 hours more to cover the same distance. We need to find the speed of the train

Solution 1

we know that

Quadratic equation

$$ax^2+bx+c=0 \quad \text{where } a \neq 0$$

i) $(x+1)^2=2(x-3)$

We know that

$$(a+b)^2=a^2+b^2+2ab$$

$$x^2+2x+1=2x-6$$

Simplifying it

$$x^2+7=0$$

Since it is a form

$$ax^2+bx+c=0 \quad \text{where } a \neq 0$$

with $b=0$

So it is a quadratic equation

ii) $x^2-2x=(-2)(3-x)$

Simplifying it

$$x^2 - 2x = -6 + 2x$$

$$x^2 - 4x + 6 = 0$$

Since it is a form

$$ax^2 + bx + c = 0 \quad \text{where } a \neq 0$$

So it is a quadratic equation

$$\text{iii) } (x-2)(x+1) = (x-1)(x+3)$$

Multiplying both the factors

$$x^2 - 2x + 2 + x = x^2 + 3x - x - 3$$

Simplifying

$$-3x + 1 = 0$$

It is not of the form

$$ax^2 + bx + c = 0 \quad \text{where } a \neq 0$$

So it is not a quadratic equation

$$\text{iv) } (x-3)(2x+1) = x(x+5)$$

Multiplying both the factors

$$2x^2 + x - 6x - 3 = x^2 + 5x$$

Simplifying

$$x^2 - 10x - 3 = 0$$

Since it is a form

$$ax^2 + bx + c = 0 \quad \text{where } a \neq 0$$

So it is a quadratic equation

$$\text{v) } (2x-1)(x-3) = (x+5)(x-1)$$

Multiplying both the factors on both sides

$$2x^2 - 6x - x + 3 = x^2 - x + 5x - 5$$

$$x^2 - 11x + 8 = 0$$

Since it is a form

$$ax^2 + bx + c = 0 \quad \text{where } a \neq 0$$

So it is a quadratic equation

$$\text{vi) } x^2 + 3x + 1 = (x-2)^2$$

We know that

$$(a+b)^2 = a^2 + b^2 + 2ab$$

$$x^2 + 3x + 1 = x^2 - 4x + 4$$

$$7x - 3 = 0$$

Since it is not of form

$$ax^2 + bx + c = 0 \quad \text{where } a \neq 0$$

So it is a not quadratic equation

$$\text{vii) } (x+2)^3 = 2x(x^2-1)$$

Important formula you must have remembered in old classes

$$(a+b)^3 = a^3 + b^3 + 3ab^2 + 3a^2b$$

$$x^3 + 8 + 6x^2 + 12x = 2x^3 - 2x$$

Simplifying

$$x^3 - 6x^2 - 14x - 8 = 0$$

Since it is not of form

$$ax^2 + bx + c = 0 \quad \text{where } a \neq 0$$

So it is a not quadratic equation

$$\text{viii) } x^3 - 4x^2 - x + 1 = (x - 2)^3$$

Important formula you must have remembered in old classes

$$(a-b)^3 = a^3 - b^3 + 3ab^2 - 3a^2b$$

$$x^3 - 4x^2 - x + 1 = x^3 - 8 - 6x^2 + 12x$$

Simplifying

$$2x^2 - 13x + 9 = 0$$

Since it is a form

$$ax^2 + bx + c = 0 \quad \text{where } a \neq 0$$

So it is a quadratic equation

Solution 2

i) Let the breadth of the plot = x m

As per given condition in the question

$$\text{Length} = 2x + 1$$

Now we know that Area is given by

$$A = LB$$

$$A = 528 \text{ m}^2$$

So

$$528 = (2x + 1)x$$

$$x^2 + 2x - 528 = 0$$

This is a quadratic equation

ii) let the two consecutive positive integers are x and $x + 1$

The product of these would be

$$x(x + 1)$$

It is given that product is 306

So

$$x(x + 1) = 306$$

$$x^2 + x - 306 = 0$$

This is a quadratic equation

iii) Let Rohan present age = x year

Then Rohan Mother present age would = $x + 26$

After 3 year,

Rohan age would be $=x+3$

Rohan mother's age would be $=x+26+3=x+29$

According to question, The product of their ages (in years) $=360$

Then

$$(x+3)(x+29)=360$$

Simplifying

$$x^2 + 29x + 3x + 87 = 360$$

$$x^2 + 32x - 273 = 0$$

This is a quadratic equation

iv) Let the speed of the train is x km/hr

Now distance travelled by the train $=480$ km

Few important formula here

Speed = Distance/time

Or Time = Distance / Speed

Case I

Time taken to travel 480 km by train will be $=480/x$

Case II

Now the speed of the train is reduced by 8 km/hr,

So speed would $(x-8)$

Now Time taken to travel 480 km will be $=480/x-8$

Now as per the question

$$480/(x-8) - 480/x = 3$$

$$[480x - 480(x-8)]/x(x-8) = 3$$

$$480x - 480x + 3840 = 3x(x-8)$$

$$3x^2 - 24x - 3840 = 0$$

This is a quadratic equation