Quadratic Equations Exercise-3

2	Square method	In this method we create square on LHS and RHS and then find the value. $ax^2 + bx + c = 0$ 1) $x^2 + (b/a)x + (c/a) = 0$ 2) $(x + b/2a)^2 - (b/2a)^2 + (c/a) = 0$ 3) $(x + b/2a)^2 = (b^2 - 4ac)/4a^2$ 4) $x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$ Example $x^2 + 4x - 5 = 0$ 1) $(x+2)^2 - 4 - 5 = 0$ 2) $(x+2)^2 = 9$ 3) Roots of the equation can be find using square root on both the sides $x+2 = -3 \Rightarrow x = -5$ $x+2 = 3 \Rightarrow x = 1$
3	Quadratic method	For quadratic equation $ax^2 + bx + c = 0$, roots are given by $x = \frac{-b+\sqrt{b^2-4ac}}{2a}$, $x = \frac{-b-\sqrt{b^2-4ac}}{2a}$ For $b^2 - 4ac > 0$, Quadratic equation has two real roots of

		different value For b^2-4ac =0, quadratic equation has one real root For b^2-4ac < 0, no real roots for quadratic equation

Question 1

Find the roots of the following quadratic equations, if they exist, by the method of completing the square:

(i) $2x^2 - 7x + 3 = 0$

(ii) $2x^2 + x - 4 = 0$

(iii) $4x^2 + 4\sqrt{3}x + 3 = 0$

(iv) $2x^2 + x + 4 = 0$

Answer

(i) $2x^2 - 7x + 3 = 0$

On dividing both sides of the equation by 2, we get

$x^2 - 7x/2 \quad +3/2=0$

$x^2 - 2 \times x \times \ 7/4 \ +3/2=0$

adding $(7/4)^2$ and subtracting on LHS, we get

$(x)^2 - 2 \times x \times 7/4 + (7/4)^2 - (7/4)^2 + 3/2=0$

$(x - 7/4)^2 = 49/16 - 3/2$

$(x - 7/4)^2 = 25/16$

$(x - 7/4) = \pm 5/4$

$x = 7/4 \pm 5/4$

$x = 7/4 + 5/4$ or $x = 7/4 - 5/4$

$x = 12/4$ or $x = 2/4$

$x = 3$ or $1/2$

(ii) $2x^2 + x - 4 = 0$

On dividing both sides of the equation, we get

$x^2 + x/2 - 2=0$

adding $(1/4)^2$ and Subtracting to LHS, we get

$(x)^2 + 2 \times x \times 1/4 + (1/4)^2 - (1/4)^2 -2 =0$

$(x + 1/4)^2 = 33/16$

$\Rightarrow x + 1/4 = \pm \sqrt{33}/4$

$\Rightarrow x = \pm \sqrt{33}/4 - 1/4$

$\Rightarrow x = \pm \sqrt{33}-1/4$

$\Rightarrow x = \sqrt{33}-1/4$ or $x = -\sqrt{33}-1/4$

(iii) $4x^2 + 4\sqrt{3}x + 3 = 0$

$(2x)^2 + 2 \times 2x \times \sqrt{3} + (\sqrt{3})^2 = 0$

$(2x + \sqrt{3})^2 = 0$

$(2x + \sqrt{3}) = 0$ and $(2x + \sqrt{3}) = 0$
$x = -\sqrt{3}/2$ or $x = -\sqrt{3}/2$

(iv) $2x^2 + x + 4 = 0$
On dividing both sides of the equation, we get
$x^2 + 1/2x + 2 = 0$
Adding and Subtracting $(1/4)^2$ to LHS, we get
$(x)^2 + 2 \times x \times 1/4 + (1/4)^2 - (1/4)^2 + 2 = 0$
$(x + 1/4)^2 = 1/16 - 2$
$(x + 1/4)^2 = -31/16$
However, the square of number cannot be negative.
Therefore, there is no real root for the given equation.

Question 2
Find the roots of the quadratic equations given in Q.1 above by applying the quadratic formula.

Answer

(i) $2x^2 - 7x + 3 = 0$
On comparing this equation with $ax^2 + bx + c = 0$, we get
$a = 2$, $b = -7$ and $c = 3$
By using quadratic formula

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Substituting the values, we get

$$x = \frac{7 \pm \sqrt{49 - 24}}{4}$$

$$x = \frac{7 \pm 5}{4}$$

Taking + and – separately we get
$x = 3$ or $1/2$

(ii) $2x^2 + x - 4 = 0$
On comparing this equation with $ax^2 + bx + c = 0$, we get
$a = 2$, $b = 1$ and $c = -4$
By using quadratic formula

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Substituting the values, we get
$x = -1+\sqrt{33}/4$ or $x = -1-\sqrt{33}/4$

(iii) $4x^2 + 4\sqrt{3}x + 3 = 0$
On comparing this equation with $ax^2 + bx + c = 0$, we get
$a = 4$, $b = 4\sqrt{3}$ and c = 3
By using quadratic formula

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Substituting the values, we get
$x = \sqrt{3}/2$ or $x = -\sqrt{3}/2$

(iv) $2x^2 + x + 4 = 0$
On comparing this equation with $ax^2 + bx + c = 0$, we get
$a = 2$, $b = 1$ and $c = 4$
By using quadratic formula

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$x = -1 \pm \sqrt{1-32}/4$
$x = -1 \pm \sqrt{-31}/4$
The square of a number can never be negative.
∴There is no real solution of this equation.

Question 2
Find the roots of the following equations:
(i) $x-1/x = 3$, $x \neq 0$
(ii) $1/x+4 - 1/x-7 = 11/30$, $x = -4, 7$

Answer

(i) $x-1/x = 3$
$\Rightarrow x^2 - 3x - 1 = 0$
On comparing this equation with $ax^2 + bx + c = 0$, we get
$a = 1$, $b = -3$ and $c = -1$
By using quadratic formula, we get
$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$x = 3\pm\sqrt{9+4}/2$
$x = 3\pm\sqrt{13}/2$
$x = 3+\sqrt{13}/2$ or $x = 3-\sqrt{13}/2$

(ii) $1/x+4 - 1/x-7 = 11/30$
$x-7-x-4/(x+4)(x-7) = 11/30$
$-11/(x+4)(x-7) = 11/30$
$(x+4)(x-7) = -30$
$x^2 - 3x - 28 = 30$
$x^2 - 3x + 2 = 0$
By using quadratic formula, we get
$$x = \frac{-b\pm\sqrt{b^2-4ac}}{2a}$$
Substituting the values
$x = 1$ or 2

Question 4
The sum of the reciprocals of Rehman's ages, (in years) 3 years ago and 5 years from now is 1/3. Find his present age.

Answer

Let the present age of Rehman be x years.
Three years ago, his age was $(x - 3)$ years.
Five years hence, his age will be $(x + 5)$ years.
As per question, the sum of the reciprocals of Rahman's ages 3 years ago and 5 years from now is 1/3.
$\therefore 1/(x-3) + 1/(x-5) = 1/3$
$(x+5+x-3)/(x-3)(x+5) = 1/3$
$3(2x + 2) = (x-3)(x+5)$
$x^2 - 4x - 21 = 0$
Solving as per Factoring method
$x^2 - 7x + 3x - 21 = 0$
$x(x - 7) + 3(x - 7) = 0$
$(x - 7)(x + 3) = 0$
$x = 7, -3$
However, age cannot be negative.
Therefore, Rehman's present age is 7 years.

Question 5
In a class test, the sum of Shefali' s marks in Mathematics and English is 30. Had she got 2 marks more in Mathematics and 3 marks less in English, the product of their marks would have been 210. Find her marks in the two subjects.

Answer

Let the marks in Mathematics be x.

Then, the marks in English will be 30 - x.
According to the question,
$(x + 2)(30 - x - 3) = 210$
$(x + 2)(27 - x) = 210$
$-x^2 + 25x + 54 = 210$
$x^2 - 25x + 156 = 0$
$x^2 - 12x - 13x + 156 = 0$
$x(x - 12) - 13(x - 12) = 0$
$(x - 12)(x - 13) = 0$
$x = 12, 13$
If the marks in Mathematics are 12, then marks in English will be 30 - 12 = 18
If the marks in Mathematics are 13, then marks in English will be 30 - 13 = 17

Question 6
The diagonal of a rectangular field is 60 meters more than the shorter side. If the longer side is 30 meters more than the shorter side, find the sides of the field.

Answer

Let the shorter side of the rectangle be x m.
Then, larger side of the rectangle = $(x + 30)$ m

Now Diagonal of the rectangle is given by
$D = \sqrt{l^2 + b^2}$
$D = \sqrt{x^2 + (x + 30)^2}$
Now D=(x+30)
So
$x + 60 = \sqrt{x^2 + (x + 30)^2}$
Squaring both the sides
$x^2 + (x + 30)^2 = (x + 60)^2$
$x^2 - 60x - 2700 = 0$\
$x^2 - 90x + 30x - 2700 = 0$
$x(x - 90) + 30(x - 90)$
$(x - 90)(x + 30) = 0$
$x = 90, -30$
However, side cannot be negative. Therefore, the length of the shorter side will be 90 m.
Hence, length of the larger side will be (90 + 30) m = 120 m.

Question 7
The difference of squares of two numbers is 180. The square of the smaller number is 8 times the larger number. Find the two numbers.

Answer

Let the larger and smaller number be x and y respectively.
According to the question,
$x^2 - y^2 = 180$ ---(A)
$y^2 = 8x$ ---(B)
From equation A and B
$x^2 - 8x = 180$
$x^2 - 8x - 180 = 0$
$x = 18, -10$

However, the larger number cannot be negative as 8 times of the larger number will be negative and hence, the square of the smaller number will be negative which is not possible.
Therefore, the larger number will be 18 only.
$x = 18$
$\therefore y^2 = 8x = 8 \times 18 = 144$
$\Rightarrow y = \pm\sqrt{44} = \pm 12$
$\therefore$ Smaller number = ± 12
Therefore, the numbers are 18 and 12 or 18 and - 12.

Question 8
A train travels 360 km at a uniform speed. If the speed had been 5 km/h more, it would have taken 1 hour less for the same journey. Find the speed of the train.

Answer

Let the speed of the train be x km/hr.
Time taken to cover 360 km = $360/x$ hr.
According to the question,
$(x + 5)\ [360-(1/x)] = 360$
$360 - x + 1800-(5/x) = 360$
$x^2 + 5x + 10x - 1800 = 0$
$x = 40, -45$
However, speed cannot be negative.
Therefore, the speed of train is 40 km/h.

Question 9
Two water taps together can fill a tank in 75/8 hours. The tap of larger diameter takes 10 hours less than the smaller one to fill the tank separately. Find the time in which each tap can separately fill the tank.

Answer

Let the time taken by the smaller pipe to fill the tank be x hr.
Time taken by the larger pipe = $(x - 10)$ hr.
Part of tank filled by smaller pipe in 1 hour = $1/x$
Part of tank filled by larger pipe in 1 hour = $1/x - 10$

It is given that the tank can be filled in 75/8 hours by both the pipes together. Therefore,

$1/x + 1/(x-10) = 8/75$

$(x-10+x)/x(x-10) = 8/75$

$2x-10/x(x-10) = 8/75$

$75(2x - 10) = 8x^2 - 80x$

$8x^2 - 230x + 750 = 0$

$x = 25, 30/8$

Time taken by the smaller pipe cannot be 30/8 = 3.75 hours. As in this case, the time taken by the larger pipe will be negative, which is logically not possible.

Therefore, time taken individually by the smaller pipe and the larger pipe will be 25 and 25 - 10 =15 hours respectively.

Question 10

An express train takes 1 hour less than a passenger train to travel 132 km between Mysore and Bangalore (without taking into consideration the time they stop at intermediate stations). If the average speed of the express train is 11 km/h more than that of the passenger train, find the average speed of the two trains.

Answer

Let the average speed of passenger train be x km/h.

Average speed of express train = $(x + 11)$ km/h

It is given that the time taken by the express train to cover 132 km is 1 hour less than the passenger train to cover the same distance.

$$\frac{132}{x} - \frac{132}{x+11} = 1$$

$$132\left[\frac{x+11-x}{x(x+11)}\right] = 1$$

$132 \times 11 = x(x + 11)$

$x^2 + 11x - 1452 = 0$

$x = -44, 33$

Speed cannot be negative.

Therefore, the speed of the passenger train will be 33 km/h and thus, the speed of the express train will be 33 + 11 = 44 km/h.

Question 11

Sum of the areas of two squares is 468 m^2. If the difference of their perimeters is 24 m, find the sides of the two squares.

Answer

Let the sides of the two squares be x m and y m. Therefore, their perimeter will be $4x$ and $4y$ respectively and their areas will be x^2 and y^2 respectively.

As per question
$4x - 4y = 24$
$x - y = 6$
$x = y + 6$
Also, $x^2 + y^2 = 468$
So $(6 + y^2) + y^2 = 468$
$2y^2 + 12y + 432 = 0$
$y^2 + 6y - 216 = 0$
Solving by factor method
$y = -18, 12$
However, side of a square cannot be negative.
Hence, the sides of the squares are 12 m and (12 + 6) m = 18 m.