Quadratic Equations Exercise-4

Nature of roots of Quadratic equation

S.no	Condition	Nature of roots
1	$b^2 -4ac > 0$	Two distinct real roots
2	$b^2-4ac =0$	One real root
3	$b^2-4ac < 0$	No real roots

Question 1

Find the nature of the roots of the following quadratic equations. If the real roots exist, find them;

(i) $2x^2 - 3x + 5 = 0$

(ii) $3x^2 - 4\sqrt{3}x + 4 = 0$

(iii) $2x^2 - 6x + 3 = 0$

Answer

(i) Consider the equation

$x^2 - 3x + 5 = 0$

Comparing it with $ax^2 + bx + c = 0$, we get

$a = 2$, $b = -3$ and $c = 5$

Discriminant = $b^2 - 4ac$

$= (- 3)^2 - 4 (2) (5) = 9 - 40$

$= - 31$

As $b^2 - 4ac < 0$,

Therefore, no real root is possible for the given equation.

(ii) $3x^2 - 4\sqrt{3}x + 4 = 0$

Comparing it with $ax^2 + bx + c = 0$, we get

$a = 3$, $b = -4\sqrt{3}$ and $c = 4$

Discriminant = $b^2 - 4ac$

$= (-4\sqrt{3})^2 - 4(3)(4)$

$= 48 - 48 = 0$

As $b^2 - 4ac = 0$,

Therefore, real roots exist for the given equation and they are equal to each other.

And the roots will be $-b/2a$ and $-b/2a$.$-b/2a = -(-4\sqrt{3})/2\times3 = 4\sqrt{3}/6 = 2\sqrt{3}/3 = 2/\sqrt{3}$
Therefore, the roots are $2/\sqrt{3}$ and $2/\sqrt{3}$.

(iii) $2x^2 - 6x + 3 = 0$
Comparing this equation with $ax^2 + bx + c = 0$, we get
$a = 2, b = -6, c = 3$
Discriminant $= b^2 - 4ac$
$= (-6)^2 - 4 (2) (3)$
$= 36 - 24 = 12$
As $b^2 - 4ac > 0$,
Therefore, distinct real roots exist for this equation:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Substituting the values of a, b, c,
$x = (3+\sqrt{3})/2$ or $(3-\sqrt{3})/2$

Question 2
Find the values of k for each of the following quadratic equations, so that they have two equal roots.
(i) $2x^2 + kx + 3 = 0$
(ii) $kx (x - 2) + 6 = 0$

Answer

(i) $2x^2 + kx + 3 = 0$
Comparing equation with $ax^2 + bx + c = 0$, we get
$a = 2$, $b = $ k and $c = 3$
Discriminant $= b^2 - 4ac$
$= (k)^2 - 4(2) (3)$
$= k^2 - 24$
For equal roots,
Discriminant = 0
$k^2 - 24 = 0$
$k^2 = 24$
$k = \pm\sqrt{24} = \pm2\sqrt{6}$

(ii) $kx(x - 2) + 6 = 0$
or $kx^2 - 2kx + 6 = 0$
Comparing this equation with $ax^2 + bx + c = 0$, we get
$a = k$, $b = - 2k$ and $c = 6$
Discriminant $= b^2 - 4ac$
$= (- 2k)2 - 4 (k) (6)$
$= 4k2 - 24k$
For equal roots,
$b2 - 4ac = 0$

$4k2 - 24k = 0$
$4k(k - 6) = 0$
Either $4k = 0$
or $k = 6 = 0$
$k = 0$ or $k = 6$
However, if $k = 0$, then the equation will not have the terms 'x^2' and 'x'.
Therefore, if this equation has two equal roots, k should be 6 only.

Question 3
Is it possible to design a rectangular mango grove whose length is twice its breadth, and the area is 800 m^2? If so, find its length and breadth.

Answer

Let the breadth of mango grove be x.
Length of mango grove will be $2x$.
Area of mango grove = $(2x)(x) = 2x^2$
$2x^2 = 800$
$x^2 = 800/2 = 400$
$x^2 - 400 = 0$
Comparing this equation with $ax^2 + bx + c = 0$, we get
$a = 1$, $b = 0$, $c = 400$
Discriminant = $b^2 - 4ac$
= 1600
Here, $b^2 - 4ac > 0$
Therefore, the equation will have real roots. And hence, the desired rectangular mango grove can be designed.
$x = \pm 20$
However, length cannot be negative.
Therefore, breadth of mango grove = 20 m
Length of mango grove = $2 \times 20 = 40$ m

Question 4
Is the following situation possible? If so, determine their present ages. The sum of the ages of two friends is 20 years. Four years ago, the product of their ages in years was 48.

Answer

Let the age of one friend be x years.
then the age of the other friend will be (20 - x) years.
4 years ago,
Age of 1st friend = (x - 4) years
Age of 2nd friend = (20 - x - 4) = (16 - x) years
A/q we get that,
$(x - 4)(16 - x) = 48$

$16x - x^2 - 64 + 4x = 48$
$- x^2 + 20x - 112 = 0$
$x^2 - 20x + 112 = 0$
Comparing this equation with $ax^2 + bx + c = 0$, we get
$a = 1$, $b = -20$ and $c = 112$
Discriminant = $b^2 - 4ac$
$= (-20)^2 - 4 \times 112$
$= 400 - 448 = -48$
$b^2 - 4ac < 0$
As discriminant is negative, there will be no real solution possible for the equations. Such type of condition doesn't exist.

Question 5
Is it possible to design a rectangular park of perimeter 80 and area 400 m^2? If so find its length and breadth.

Answer

Let the length and breadth of the park be x and y.
Perimeter = $2(x + y) = 80$
$x + y = 40$
Or, $y = 40 - x$ ---(A)
Area = $x \times y$
Substituting value of y from equation A
$= x(40 - x) = 40x - x^2 = 400$
$x^2 - 40x + 400 = 0$
Comparing this equation with $ax^2 + bx + c = 0$, we get
$a = 1$, $b = -40$, $c = 400$
Discriminant = $b^2 - 4ac$
$(-40)^2 - 4 \times 400$
$= 1600 - 1600 = 0$
$b^2 - 4ac = 0$
Therefore, this equation has equal real roots. And hence, this situation is possible.
Root of this equation, $x = -b/2a$
$x = (40)/2(1) = 40/2 = 20$
Therefore, length of park, $x = 20$ m
And breadth of park, $y = 40 - a = 40 - 20 = 20$ m.