Real Numbers Exercise 1

Question 1
Use Euclid's division algorithm to find the HCF of :

(i) 135 and 225
(ii) 196 and 38220
(iii) 867 and 255

Question 2
Show that any positive odd integer is of the form $6q + 1$, or $6q + 3$, or $6q + 5$, where q is some integer.

Question 3
An army contingent of 616 members is to march behind an army band of 32 members in a parade. The two groups are to march in the same number of columns. What is the maximum number of columns in which they can march?

Question 4
Use Euclid's division lemma to show that the square of any positive integer is either of the form $3m$ or $3m + 1$ for some integer m.
[**Hint :** Let x be any positive integer then it is of the form $3q$, $3q + 1$ or $3q + 2$. Now square each of these and show that they can be rewritten in the form $3m$ or $3m + 1$.]

Question 5
Use Euclid's division lemma to show that the cube of any positive integer is of the form $9m$, $9m + 1$ or $9m + 8$

Solution 1
We know two things from the chapter
Euclid's Division Lemma
For a and b any two positive integer, we can always find unique integer q and r such that
$a=bq + r \quad , \quad 0 \le r \le b$
If r =0, then b is divisor of a.
HCF (Highest common factor)
HCF of two positive integers can be find using the Euclid's Division Lemma algorithm
We know that for any two integers a, b. we can write following expression
$a=bq + r \quad , \quad 0 \le r \le b$
If r=0 ,then
HCF(a,b) =b

If r≠0 , then

HCF (a,b) = HCF (b,r)

Again expressing the integer b,r in Euclid's Division Lemma, we get

$b=pr + r_1$

HCF (b,r)=HCF (r,r_1)

Similarly successive Euclid 's division can be written until we get the remainder zero, the divisor at that point is called the HCF of the a and b

We will use the same in this question

We should always start with Larger number

So in this case the larger number is 225

Now we can write 135 and 225 in Euclid division algorithm

225=135 X 1 +90

Now HCF of numbers (225,135) = HCF (135,90)

Again writing 135,90 is Euclid division formula

135=90X1 +45

Now HCF (135,90)= HCF (90,45)

Again writing 90,45 is Euclid division formula

90=45X2 +0

Now r =0, 45 is HCF (90,45)

45 is HCF (225,135)

We should always start with Larger number

So in this case the larger number is 38220

Now we can write 38220 and 196 in Euclid division algorithm

38220=196 X 195 +0

Now r =0, 196 is HCF (196,38220)

We should always start with Larger number

So in this case the larger number is 867

Now we can write 867 and 255 in Euclid division algorithm

867=255 X 3 +102

Now HCF of numbers (867,255) = HCF (255,102)

Again writing 255,102 is Euclid division formula

255=102X2 +51

Now HCF (255,102)= HCF (102,51)

Again writing 102,51 is Euclid division formula

102=51X2 +0

Now r =0, 51 is HCF (102,51)

51 is HCF (867,255)

Solution 2:

Euclid's Division Lemma

For a and b any two positive integer, we can always find unique integer q and r such that
$a=bq+r \quad , \quad 0 \le r < b$

Now on putting b=6 ,we get
$a=6q+r \quad , 0 \le r < 6$
a=6q , This is an even number
a=6q+1 , This is an odd number
a=6q+2 ,This is an even number as 6 and 2 are divisible by 2
a=6q+3, it is not divisible by 2
a=6q+4, it is divisible by 2
a=6q+5 , it is not divisible by 2
So 6q,6q+2,6q +4 are even number
6q+1,6q+3,6q+5 are odd numbers

Solution 3:
According to the questions, we need to find the maximum number of column
Maximum number of column is the HCF of number 32 and 616
So question has reduced to finding the HCF of 32 and 616 using Euclid division algorithm

We should always start with Larger number
So in this case the larger number is 616
Now we can write 616 and 32 in Euclid division algorithm
616=32 X19 +8
Now HCF of numbers (616,32) = HCF (32,8)
Again writing 32,8 is Euclid division formula
32=8X4+0
As r=0, 8 is the HCF of 616 and 32

Solution 4:
<u>Euclid's Division Lemma</u>
For a and b any two positive integer, we can always find unique integer q and r such that
$a=bq+r \quad , \quad 0 \le r < b$

Now on putting b=3 ,we get
$a=3q+r \quad , 0 \le r < 3$
$a=3q$, $a^2=9q^2$ $a^2=3m$ where $m=3q^2$
$a=3q+1$, $a^2=9q^2+6q+1=3(3q^2+2q)+1=3m+1$ where $m=3q^2+2q$
$a=3q+2$, $a^2=9q^2+12q+4=3(3q^2+4q+1)+1=3m+1$ where $m=3q^2+4q+1$

Solution 5:
<u>Euclid's Division Lemma</u>

For a and b any two positive integer, we can always find unique integer q and r such that

$a=bq + r \quad , \quad 0 \le r < b$

Now on putting b=3 ,we get

$a=3q+ r \quad , 0 \le r < 3$

$a=3q$, $a^3=27q^3$ $a^3=9m$ where $m=3q^3$

$a=3q+1$, $a^3=27q^3 +27q^2 +9q +1=9(3q^3+3q+q) +1 =9m+1$ where $m=3q^3+3q+q$

$a=3q+2$, $a^2=27q^3 +54q^2 +36q +8=9(3q^3+6q+4q) +1 =9m+1$ where $m=3q^3+6q+4q$