Quadrilaterals Exercise 1

The question is these will extensively Triangle congruence to prove various facts

I am giving here the short summary of what we learned in Triangles

<table>
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<th>N</th>
<th>Criterion</th>
<th>Description</th>
<th>Figures and expression</th>
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| 1  | Side angle Side (SAS) congruence | • Two triangles are congruent if the two sides and included angles of one triangle is equal to the two sides and included angle  
• It is an axiom as it cannot be proved so it is an accepted truth  
• ASS and SSA type two triangles may not be congruent always | ![](image1)  
If following condition  
AB=DE, BC=EF  
∠B = ∠E  
Then  
ABC ≅ DEF |
| 2  | Angle side angle (ASA) congruence | • Two triangles are congruent if the two angles and included side of one triangle is equal to the corresponding angles and side  
• It is a theorem and can be proved | ![](image2) |
<table>
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<th>Angle angle side (AAS) congruence</th>
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If following condition

BC = EF

∠B = ∠E, ∠C = ∠F

Then

ABC  ≅  DEF

If following condition

BC = EF

∠A = ∠D, ∠C = ∠F

Then

ABC  ≅  DEF

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Question 1:
The angles of quadrilateral are in the ratio 3 : 5 : 9 : 13. Find all the angles of the Quadrilateral

Solution:
Let the common ratio between the angles be $y$. Therefore, the angles will be $3y$, $5y$, $9y$, and $13y$ respectively.

As the sum of all interior angles of a quadrilateral is $360^\circ$

$3y + 5y + 9y + 13y = 360^\circ$

$30y = 360^\circ$

$y = 12^\circ$

Hence, the angles are

$3y = 3 \times 12 = 36^\circ$

$5y = 5 \times 12 = 60^\circ$

$9y = 9 \times 12 = 108^\circ$

$13y = 13 \times 12 = 156^\circ$

**Question 2**

If the diagonals of a parallelogram are equal, then show that it is a rectangle

**Solution:**

Let ABCD is a parallelogram

![Parallelogram](image)

**Given**

$AC = BD$

$AD = BC$ and $CD = AB$ (in parallelogram opposite side are equal)

**To Prove:**

Show that it is a rectangle

**Proof**
In $\triangle$ ADC and $\triangle$ BCD

$AD = BC$

$CD = CD$

$AC = BD$

So by SSS congruence

$\triangle$ ADC $\cong$ $\triangle$ BCD

So $\angle$ ADC = $\angle$ BCD (by CPCT) (corresponding parts of the two congruent triangles)

But, we also have $\angle$ ADC + $\angle$ BCD = 180° (Co-interior angles because BC || AD)

So 2$\angle$ ADC = 180

=> $\angle$ ADC = 90

So $\angle$ BCD = 90

So all the angles A, B, C, D are 90°. Hence rectangle

Question 3:

Show that if the diagonals of a quadrilateral bisect each other at right angles, then it is a rhombus.

Solution:

Given

Let ABCD be the quadrilateral and AC and BD are diagonal which bisect at right angles

OA = OC, OB = OD and $\angle$ AOB = $\angle$ BOC = $\angle$ COD = $\angle$ AOD = 90°
**To Prove:** ABCD is rhombus

**Proof:**

Now, in $\triangle AOD$ and $\triangle COD$

$OA = OC$ (Diagonal bisects each other)

$\angle AOD = \angle COD$ (given)

$OD = OD$ (common)

$\triangle AOD \cong \triangle COD$ (by SAS congruence rule)

$\therefore AD = CD$ by CPCT (1)

Similarly we can prove that

$AD = AB$ and $CD = BC$ (2)

From equations (1) and (2), we can say that

$AB = BC = CD = AD$

Since all sides are equal, it is a rhombus

**Question 4:**
Show that the diagonals of a square are equal and bisect each other at right angles.

**Solution:**

Let $ABCD$ is a square whose diagonal $BD$ and $AC$ intersect at $O$

**Given**

$AB = BC = CD = AD$

$\angle A = \angle B = \angle C = \angle D = 90$
To prove: Diagonal are equal and bisect each other at right angle

OA=OC
OD=OB
AC=BD

\( AC \perp BD \)

Proof:

In \( \triangle ABC \) and \( \triangle DCB \),
- \( AB = DC \) (From Sides of a square are equal to each other)
- \( \angle ABC = \angle DCB \) (All interior angles are of 90)
- \( BC = CB \) (Common side)
- \( \triangle ABC \cong \triangle DCB \) (By SAS congruency)
- \( AC = DB \) (By CPCT)

Hence, the diagonals of a square are equal in length.

In \( \triangle AOB \) and \( \triangle COD \),
- \( \angle AOB = \angle COD \) (Vertically opposite angles)
- \( \angle ABO = \angle CDO \) (Alternate interior angles)
- \( AB = CD \) (Sides of a square are always equal)

\( \therefore \) \( \triangle AOB \cong \triangle COD \) (By AAS congruence rule)

\( \therefore \) \( AO = CO \) and \( OB = OD \) (By CPCT)

Hence, the diagonals of a square bisect each other.

In \( \triangle AOB \) and \( \triangle COB \),
- As we had proved that diagonals bisect each other, therefore,
- \( AO = CO \)
- \( AB = CB \) (Sides of a square are equal)
- \( BO = BO \) (Common)

\( \therefore \) \( \triangle AOB \cong \triangle COB \) (By SSS congruency)

\( \therefore \) \( \angle AOB = \angle COB \) (By CPCT)

However, \( \angle AOB + \angle COB = 180^\circ \) (Linear pair)

\( 2\angle AOB = 180^\circ \)
\( \angle AOB = 90^\circ \)

Hence, the diagonals of a square bisect each other at right angles.
Question 5:

Show that if the diagonals of a quadrilateral are equal and bisect each other at right angles, then it is a square

Solution:

Let ABCD is a quadrilateral whose diagonal BD and AC bisect each other at right angle at O

Given

AO=OC, BO=OD, AC=BD

To Prove: ABCD is a square

Proof:

In $\triangle AOB$ and $\triangle COD$

AO=CO (Given)

$\angle AOB=\angle COD$ (Each given equal to $90^\circ$)

BO=DO (Given)

Therefore, by SAS congruence rule, $\triangle AOB \cong \triangle COD$.

$\Rightarrow \angle OBA=\angle ODC$ (by CPCT)

But, these are alternate interior angles which means that $AB \parallel CD$. (1)

Similarly, we can prove that $BC \parallel AD$. (2)

From (1) and (2), we can say that quadrilateral ABCD is a parallelogram. Hence, we have $AB=CD$ and $BC=AD$ because opposite sides of a parallelogram are equal. (3)

Now, in $\triangle AOB$ and $\triangle AOD$

AO=AO (Common)

$\angle AOB=\angle AOD$ (Each given equal to $90^\circ$)

OB=OD (Given)

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Therefore, by SAS congruence rule, we have $\triangle AOB \cong \triangle AOD$
\[ \Rightarrow AB = AD \quad \text{(by CPCT)} \quad (4) \]

In $\triangle ACD$ and $\triangle BDC$
AC=BD \quad \text{(Given)}
AD=BC \quad \text{(Proved above in (1))}
CD=DC \quad \text{(Common)}
Therefore, by SSS congruence rule, $\triangle ACD \cong \triangle BDC$

\[ \Rightarrow \angle ADC = \angle BCD \quad \text{(Corresponding parts of congruent triangles are equal)} \quad (5) \]
But, we also have $\angle ADC + \angle BCD = 180^\circ$ \quad \text{(Co-Interior angles)} \quad (6)

From (5) and (6), we can say that
$\angle ADC + \angle ADC = 180^\circ$
\[ \Rightarrow 2\angle ADC = 180^\circ \]
\[ \Rightarrow \angle ADC = 180^\circ / 2 = 90^\circ \quad (7) \]

From (3), (4) and (7), we can say that $ABCD$ is a parallelogram having all the sides equal and we have showed that it's one angle is equal to $90^\circ$ which is enough to consider it a square. Therefore, $ABCD$ is a square.

**Question 6:**

Diagonal AC of a parallelogram $ABCD$ bisects $\angle A$ (see the given figure). Show that

(i) It bisects $\angle C$ also,

(ii) $ABCD$ is a rhombus.

\[\text{Solution:}\]

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i)

Given
ABCD is a parallelogram and AC bisect angle A
∠ DAC = ∠ BAC

To Prove: AC bisects ∠ C.
Proof:
As ABCD is a parallelogram

∠ DAC = ∠ BCA (Alternate interior angles) ... (1)
And ∠ BAC = ∠ DCA (Alternate interior angles) ... (2)
But it is given that AC bisects ∠ A.
∴ ∠ DAC = ∠ BAC ... (3)
From equations (1), (2) and (3), we have
∠ DAC = ∠ BCA = ∠ BAC = ∠ DCA ... (4)
⇒ ∠ DCA = ∠ BCA
Hence, AC bisects ∠ C.

(ii)

To Prove: ABCD is a rhombus
Proof:
From equation (4), we have
∠ DAC = ∠ DCA
∴ DA = DC (side opposite to equal angles are equal)
But DA = BC and AB = CD (opposite sides of parallelogram)
∴ AB = BC = CD = DA
Hence, ABCD is rhombus

Question 7:
ABCD is a rhombus. Show that diagonal AC bisects ∠ A as well as ∠ C and diagonal BD bisects ∠ B as well as ∠ D.

Solution:

Given: ABCD is a rhombus
To Prove: Diagonal BD bisect angle B and D
Diagonal AC bisect angle A and C

Proof:
Let us join AC.
In △ ABC,
BC = AB         (Sides of a rhombus are equal to each other)
∴ ∠ BAC = ∠ BCA (Angles opposite to equal sides of a triangle are equal)
However, ∠ BAC = ∠ DCA (Alternate interior angles for parallel lines AB and CD)
⇒ ∠ BCA = ∠ DCA
Therefore, AC bisects ∠ C.
Also, ∠ BCA = ∠ DAC (Alternate interior angles for || lines BC and DA)
⇒ ∠ BAC = ∠ DAC
Therefore, AC bisects ∠ A.
Similarly, it can be proved that BD bisects ∠ B and ∠ D as well.

Question 8:
ABCD is a rectangle in which diagonal AC bisects ∠ A as well as ∠ C. Show that:

(i) ABCD is a square
(ii) Diagonal BD bisects ∠ B as well as ∠ D.

Solution

Given

This material is created by http://physicscatalyst.com/ and is for your personal and non-commercial use only.
ABCD is a rectangle and $\angle DAC = \angle BAC$ and $\angle DCA = \angle BCA$ (i)

To Prove: ABCD is a square

Proof:

In $\triangle ADC$ and $\triangle ABC$
$\angle CAD = \angle CAB$ (Given)
$AC = AC$ (Common)
$\angle DCA = \angle BCA$ (Given)

Therefore, by ASA congruence rule, $\triangle ADC \cong \triangle ABC$

$\Rightarrow AD = AB$ (by CPCT) (2)

From (1) and (2), we can say that ABCD is a rectangle having all the sides equal. It means that ABCD is a square.

(i) To prove diagonal BD bisects $\angle B$ as well as $\angle D$.

In solution (i), we have showed that ABCD is a square.

Now in $\triangle CBD$ and $\triangle ABD$
$BC = BA$ (Sides of square are equal)
$BD = BD$ (Common)
$CD = AD$ (Sides of square are equal)

Therefore, by SSS congruence rule, $\triangle CBD \cong \triangle ABD$

$\Rightarrow \angle CBD = \angle ABD$ (by CPCT) (3)

And, $\angle CDB = \angle ADB$ (by CPCT) (4)

From (3) and (4), we can say that BD bisects $\angle B$ as well as $\angle D$. 

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