Complex numbers Exercise 1

Question 1:

Express the given complex number in the form a + ib: (5i)(-3i/5)

Question 2:

Express the given complex number in the form a + ib: $i^9 + i^{19}$

Question 3:

Express the given complex number in the form a + ib: i^{-39}

Question 4:

Express the given complex number in the form a + ib: 3(7 + i7) + i(7 + i7)

Question 5:

Express the given complex number in the form a + ib: (1 – i) – (–1 + i6)

Question 6:

Express the given complex number in the form a + ib:

(1/5+2i/5)-(4+i5/2)

Question 7:

Express the given complex number in the form a + ib:

$$\left[\left(\frac{1}{3}+i\frac{7}{3}\right)+\left(4+i\frac{1}{3}\right)\right]-\left(\frac{-4}{3}+i\right)$$

Question 8

Express the given complex number in the form a + ib: $(1 - i)^4$

Question 9:

Express the given complex number in the form a + ib: $(1/3+3i)^3$

Question 10

Express the given complex number in the form a + ib: $(-2-1i/3)^3$

Question 11:

Find the multiplicative inverse of the complex number 4 – 3i.

Question 12:

Find the multiplicative inverse of the complex number

$\sqrt{5}$+3i

Question 13

Find the multiplicative inverse of the complex number –i

Question 14

Express the following expression in the form of a + ib.

$$\frac{(3+i\sqrt{5})(3-i\sqrt{5})}{(\sqrt{3}+i\sqrt{2}\,)-(\sqrt{3}-i\sqrt{2})}$$

Solution 1

(5i)(-3i/5)

Multiplying

=-(15/5)i^2

Now we know that

i^2=-1

so

=3

Solution 2

$i^9 + i^{19}$

$=i^{2X4+1} + i^{4X4+3}$

$=(i^2)^4 \, I + (i^4)^4 \, i^3$

Now $i^2=-1$ so $i^4=1$ and $i^3=-i$

$=i+(-i)$

$=i-i=0$

Solution 3

$$i^{-39} = i^{-4\times9-3} = \left(i^4\right)^{-9} \cdot i^{-3}$$
$$= (1)^{-9} \cdot i^{-3} \qquad \left[i^4 = 1\right]$$
$$= \frac{1}{i^3} = \frac{1}{-i} \qquad \left[i^3 = -i\right]$$
$$= \frac{-1}{i} \times \frac{i}{i}$$
$$= \frac{-i}{i^2} = \frac{-i}{-1} = i \qquad \left[i^2 = -1\right]$$

Solution 4:

3(7 + i7) + i(7 + i7)

$=21+21i+7i+7i^2$

Now $i^2=-1$

So

=21+28i-7

=14+28i

Solution 5:

(1 – i) – (–1 + i6)

=2-7i

Solution 6:

(1/5+2i/5)-(4+i5/2)

=[(1/5)-4] +i[(2/5) –(5/2)]

=(-19/5) +(-21/10)i

Solution 7:

$$\left[\left(\frac{1}{3}+i\frac{7}{3}\right)+\left(4+i\frac{1}{3}\right)\right]-\left(\frac{-4}{3}+i\right)$$

Opening the bracket

$$=\frac{1}{3}+i\frac{7}{3}+4+i\frac{1}{3}+\frac{4}{3}-i$$

$$=\left(\frac{1}{3}+4+\frac{4}{3}\right)+i\left(\frac{7}{3}+\frac{1}{3}-1\right)$$

$$=\frac{17}{3}+i\frac{5}{3}$$

Solution 8

$(1 – i)^4$

Can be written as

=$[(1-i)^2]^2$

$=(1+i^2-2i)^2$

$=(1-1-2i)^2$

$=4i^2=-4$

Solution 9

$$\left(\frac{1}{3}+3i\right)^3=\left(\frac{1}{3}\right)^3+(3i)^3+3\left(\frac{1}{3}\right)(3i)\left(\frac{1}{3}+3i\right)$$
$$=\frac{1}{27}+27i^3+3i\left(\frac{1}{3}+3i\right)$$
$$=\frac{1}{27}+27(-i)+i+9i^2 \qquad [i^3=-i]$$
$$=\frac{1}{27}-27i+i-9 \qquad [i^2=-1]$$
$$=\left(\frac{1}{27}-9\right)+i(-27+1)$$
$$=\frac{-242}{27}-26i$$

Solution 10

$$\left(-2-\frac{1}{3}i\right)^3=(-1)^3\left(2+\frac{1}{3}i\right)^3$$
$$=-\left[2^3+\left(\frac{i}{3}\right)^3+3(2)\left(\frac{i}{3}\right)\left(2+\frac{i}{3}\right)\right]$$
$$=-\left[8+\frac{i^3}{27}+2i\left(2+\frac{i}{3}\right)\right]$$
$$=-\left[8-\frac{i}{27}+4i+\frac{2i^2}{3}\right] \qquad [i^3=-i]$$
$$=-\left[8-\frac{i}{27}+4i-\frac{2}{3}\right] \qquad [i^2=-1]$$
$$=-\left[\frac{22}{3}+\frac{107i}{27}\right]$$
$$=-\frac{22}{3}-\frac{107}{27}i$$

Solution 11

Let z=4-3i

Conjugate is given by

$\bar{z}$ =4+3i

Modulus is given by

|z|=5

Multiple inverse of any complex number z is given by

=$\frac{\bar{z}}{|z|^2} = \frac{4+3i}{25}$

Solution 12

Let z=$\sqrt{5}$+3i

Conjugate is given by

$\bar{z}$ =$\sqrt{5}$-3i

Modulus is given by

|z|=5+9=14

Multiple inverse of any complex number z is given by

=$\frac{\bar{z}}{|z|^2} = \frac{\sqrt{5}-3i}{14}$

Solution 13

Let z=-i

Conjugate is given by

$\bar{z}$ =i

Modulus is given by

|z|=1

Multiple inverse of any complex number z is given by

$=\frac{\bar{z}}{|z|^2}=\frac{-i}{1}=-i$

Solution 14

$$\frac{(3+i\sqrt{5})(3-i\sqrt{5})}{(\sqrt{3}+\sqrt{2}i)-(\sqrt{3}-i\sqrt{2})}$$

$$=\frac{(3)^2-(i\sqrt{5})^2}{\sqrt{3}+\sqrt{2}i-\sqrt{3}+\sqrt{2}i}$$

$$=\frac{9-5i^2}{2\sqrt{2}i}$$

$$=\frac{9-5(-1)}{2\sqrt{2}i}$$

$$=\frac{9+5}{2\sqrt{2}i}\times\frac{i}{i}$$

$$=\frac{14i}{2\sqrt{2}i^2}$$

$$=\frac{14i}{2\sqrt{2}(-1)}$$

$$=\frac{-7i}{\sqrt{2}}\times\frac{\sqrt{2}}{\sqrt{2}}$$

$$=\frac{-7\sqrt{2}i}{2}$$