Polynomial expression

A polynomial expression $S(x)$ in one variable $x$ is an algebraic expression in $x$ term as

$$S(x) = a_n x^n + a_{n-1} x^{n-1} + a_{n-2} x^{n-2} + \ldots + a_1 x + a_0$$

Where $a_n, a_{n-1}, \ldots, a_0$ are constant and real numbers and $a_n$ is not equal to zero

Some important points to remember

1) $a_n, a_{n-1}, a_{n-2}, \ldots, a_1, a_0$ are called the coefficients for $x^n, x^{n-1}, \ldots, x^1, x^0$
2) $n$ is called the degree of the polynomial
3) when $a_n, a_{n-1}, a_{n-2}, \ldots, a_1, a_0$ all are zero, it is called zero polynomial
4) A constant polynomial is the polynomial with zero degree, it is a constant value polynomial
5) A polynomial of one item is called monomial, two items binomial and three items as trinomial
6) A polynomial of one degree is called linear polynomial, two degree as quadratic polynomial and degree three as cubic polynomial

Zero's or roots of the polynomial

It is a solution to the polynomial equation $S(x) = 0$ i.e. a number "$a$" is said to be a zero of a polynomial if $S(a) = 0$.

If we draw the graph of $S(x) = 0$, the values where the curve cuts the X-axis are called Zeros of the polynomial

a) Linear polynomial has only one root
b) A zero polynomial has all the real number as roots
c) A constant polynomial has no zeros

Remainder Theorem's

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If \( p(x) \) is an polynomial of degree greater than or equal to 1 and \( p(x) \) is divided by the expression \((x-a)\), then the reminder will be \( p(a) \)

**Factor’s Theorem’s**

If \( x-a \) is a factor of polynomial \( p(x) \) then \( p(a)=0 \) or if \( p(a) =0 \), \( x-a \) is the factor the polynomial \( p(x) \)

**Geometric Meaning of the Zero’s of the polynomial**

Let us assume

\[ y = p(x) \]

where \( p(x) \) is the polynomial of any form.

Now we can plot the equation \( y=p(x) \) on the Cartesian plane by taking various values of \( x \) and \( y \) obtained by putting the values. The plot or graph obtained can be of any shapes.

The zero’s of the polynomial are the points where the graph meet \( x \) axis in the Cartesian plane. If the graph does not meet \( x \) axis, then the polynomial does not have any zero’s.

Let us take some useful polynomial and shapes obtained on the Cartesian plane

<table>
<thead>
<tr>
<th>S.no</th>
<th>( y=p(x) )</th>
<th>Graph obtained</th>
<th>Name of the graph</th>
<th>Name of the equation</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>( y=ax+b )  where ( a ) and ( b ) can be any values ((a\neq0))</td>
<td><img src="image" alt="Graph of y=ax+b" /></td>
<td>Straight line. It intersect the ( x )-axis at (( -b/a ,0))</td>
<td>Linear polynomial</td>
</tr>
<tr>
<td></td>
<td>Example ( y=2x+3 )</td>
<td><img src="image" alt="Graph of y=2x+3" /></td>
<td>Example ((-3/2,0))</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>( y=ax^2+bx+c ) ( \text{where} ) ( b^2-4ac &gt; 0 ) and ( a\neq0 ) and ( a&gt;0 )</td>
<td><img src="image" alt="Graph of y=ax^2+bx+c" /></td>
<td>Parabola It intersect the ( x )-axis at two points</td>
<td>Quadratic polynomial</td>
</tr>
<tr>
<td></td>
<td>Example ( y=x^2-7x+12 )</td>
<td><img src="image" alt="Graph of y=x^2-7x+12" /></td>
<td>Example ((3,0)) and ((4,0))</td>
<td></td>
</tr>
</tbody>
</table>
| 3  | $y=ax^2+bx+c$  
where 
$\Delta = b^2-4ac > 0$ and $a\neq0$ and $a < 0$  
Example 
y=$-x^2+2x+8$ | Parabola  
It intersect the $x$-axis at two points  
Example $(-2,0)$ and $(4,0)$ | Quadratic polynomial |
|---|---|---|---|
| 4  | $y=ax^2+bx+c$  
where 
$\Delta = b^2-4ac = 0$ and $a\neq0$ $a > 0$  
Example 
y=$(x-2)^2$ | Parabola  
It intersect the $x$-axis at one points | Quadratic polynomial |
| 5  | $y=ax^2+bx+c$  
where 
$\Delta = b^2-4ac < 0$ and $a\neq0$ $a > 0$  
Example 
y=$x^2-2x+6$ | Parabola  
It does not intersect the $x$-axis  
It has no zero’s | Quadratic polynomial |
| 6  | $y=ax^2+bx+c$  
where 
$\Delta = b^2-4ac < 0$ and $a\neq0$ $a < 0$  
Example 
y=$-x^2-2x-6$ | Parabola  
It does not intersect the $x$-axis  
It has no zero’s | Quadratic polynomial |
7 \[ y=ax^3+bx^2+cx+d \]

where \( a\neq 0 \)

It can be of any shape

It will cut the x-axis at the most 3 times

Cubic Polynomial

8 \[ a_nx^n + a_{n-1}x^{n-1} + a_{n-2}x^{n-2} + \ldots + ax + a_0 \]

Where \( a_n \neq 0 \)

It can be of any shape

It will cut the x-axis at the most \( n \) times

Polynomial of \( n \) degree

Relation between coefficient and zero’s of the Polynomial:

<table>
<thead>
<tr>
<th>S.no</th>
<th>Type of Polynomial</th>
<th>General form</th>
<th>Zero’s</th>
<th>Relationship between Zero’s and coefficients</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Linear polynomial</td>
<td>( ax+b, a\neq 0 )</td>
<td>1</td>
<td>( k = \frac{-\text{constant term}}{\text{Coefficient of } x} )</td>
</tr>
</tbody>
</table>
| 2    | Quadratic          | \( ax^2+bx+c, a\neq 0 \) | 2      | \( k_1 + k_2 = -\frac{\text{Coefficient of } x}{\text{Coefficient of } x^2} \)  \\
|      |                    |              |        | \( k_1k_2 = \frac{\text{Constant term}}{\text{Coefficient of } x^2} \) |
| 3    | Cubic              | \( ax^3+bx^2+cx+d, a\neq 0 \) | 3      | \( k_1 + k_2 + k_3 = -\frac{\text{Coefficient of } x^2}{\text{Coefficient of } x^3} \)  \\
|      |                    |              |        | \( k_1k_2k_3 = -\frac{\text{Constant term}}{\text{Coefficient of } x^{32}} \)  \\
|      |                    |              |        | \( k_1k_2 + k_2k_3 + k_1k_3 = \frac{\text{Coefficient of } x}{\text{Coefficient of } x^2} \) |
Formation of polynomial when the zeros are given

<table>
<thead>
<tr>
<th>Type of polynomial</th>
<th>Zero's Polynomial Formed</th>
</tr>
</thead>
<tbody>
<tr>
<td>Linear</td>
<td>k=a (x-a)</td>
</tr>
<tr>
<td>Quadratic</td>
<td>k₁=a and k₂=b (x-a)(x-b)</td>
</tr>
<tr>
<td></td>
<td>Or</td>
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<tr>
<td></td>
<td>x²-(a+b)x +ab</td>
</tr>
<tr>
<td></td>
<td>Or</td>
</tr>
<tr>
<td></td>
<td>x²-( Sum of the zero's)x +product of the zero’s</td>
</tr>
<tr>
<td>Cubic</td>
<td>k₁=a ,k₂=b and k₃=c (x-a)(x-b)(x-c)</td>
</tr>
</tbody>
</table>

Division algorithm for Polynomial

Let's p(x) and q(x) are any two polynomial with q(x) ≠0 ,then we can find polynomial s(x) and r(x) such that

P(x)=s(x) q(x) + r(x)

Where r(x) can be zero or degree of r(x) < degree of g(x)

Dividend =Quotient X Divisor + Remainder

Steps to divide a polynomial by another polynomial

1) Arrange the term in decreasing order in both the polynomial
2) Divide the highest degree term of the dividend by the highest degree term of the divisor to obtain the first term,
3) Similar steps are followed till we get the reminder whose degree is less than of divisor

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