Fourier Series

April 13, 2012

Fourier series is an expansion of a periodic function of period 2π which is representation of a function in a series of sine or cosine such as

$f(x) = a_0 + \sum_{n=1}^{\infty} a_n cos(nx) + \sum_{n=1}^{\infty} b_n sin(nx)$

where a_0 , a_n and b_n are constants and are known as fourier coefficents.

In applying fourier theorem for analysis of an complex periodic function , given function must satisfy following condition

(i) It should be single valued

(ii) It should be continous.

Drichlet's Conditions(sufficient but not necessary)

When a function $f(x)$ is to be expanded in the interval (a,b)

(a) $f(a)$ is continous in interval (a,b) except for finite number of finite discontinuties.

(b) $f(x)$ has finite number of maxima and minima in this interval.

Orthogonal property of sine and cosine functions

$\int_{-\pi}^{\pi} sin(mx)cos(mx)dx = 0$

$\int_{-\pi}^{\pi} sin(mx)sin(nx)dx = \int_{-\pi}^{\pi} sin(mx)sin(nx)dx = \begin{bmatrix} \pi\delta_{mn} & m \neq 0 \\ 0 & m = 0 \end{bmatrix}$

$\int_{-\pi}^{\pi} cos(mx)cos(nx)dx = \begin{bmatrix} \pi\delta_{mn} & m \neq 0 \\ 2\pi & m = 0 \end{bmatrix}$

Fourier Constants

$a_0 = \frac{1}{2\pi} \int_{-\pi}^{\pi} f(x)dx$ a_0 is the average value of function $f(x)$ over the interval

$a_n = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x)cos(nx)dx$ $b_n = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x)sin(nx)dx$

For even functions

$f(-x) = f(x)$ and fourier series becomes $f(x) = a_0 + \sum_{n=1}^{\infty} a_n cos(nx)$

For odd functions

$f(-x) = -f(x)$ and fourier series becomes $f(x) = a_0 + \sum_{n=1}^{\infty} a_n sin(nx)$

Complex form of fourier series

putting $c_0 = c_0$

$c_n = \frac{a_n - ib_n}{2}$

and

$c_{-n} = \frac{a_n + ib_n}{2}$ $f(x) = \sum_{-\infty}^{\infty} C_n e^{inx}$

coefficent

$C_n = \frac{1}{2\pi} \int_{-\pi}^{\pi} f(x)e^{-inx}dx$

Fourier series in interval (0,T)

General fourier series of a periodic piecewise continous function $f(T)$

having period $T = \frac{2\pi}{\omega}$ is

$f(t) = a_0 + \sum_{n=1}^{\infty} a_n cos(nx) + \sum_{n=1}^{\infty} b_n sin(nx)$

where

$a_0 = \frac{1}{T} \int_0^T f(t)dt$

$a_n = \frac{2}{T} \int_0^T f(t)cos(n\omega T)dt$

$b_n = \frac{2}{T} \int_0^T f(t)sin(n\omega T)dt$

Complex Form of Fourier Series

$f(x) = \sum_{n=-\infty}^{\infty} C_n e^{-i\omega t}$

where

$c_n = \frac{1}{T} \int_0^T f(t)e^{-i\omega t}dx$

Advantages of Fourier series

- It can also represent discontinous functions
- Even and odd functions are conveniently represented as cosine and sine series.
- Fourier expansion gives no assurance of its validity outside the interval.

Change of interval from $(-\pi, \pi)$ to $(-l, l)$

Series will be

$f(x) = a_0 + \sum_{n=1}^{\infty} a_n cos(\frac{nx\pi}{l}) + \sum_{n=1}^{\infty} b_n sin(\frac{nx\pi}{l})$

with

$a_0 = \frac{1}{2l} \int_{-l}^{l} f(x)dx$

$a_n = \frac{1}{2l} \int_{-l}^{l} f(x)cos(\frac{n\pi x}{l})dx$

$b_n = \frac{1}{2l} \int_{-l}^{l} f(x)sin(\frac{n\pi x}{l})dx$

Fourier Series in interval $(0, l)$

Cosine series when function $f(x)$ is even

$f(x) = a_0 + \sum_{n=1}^{\infty} a_n cos(\frac{n\pi x}{l})$

$a_0 = \frac{1}{l}\int_0^l f(x)dx$

$a_n = \frac{2}{l}\int_0^l f(x)cos(\frac{n\pi x}{l})dx$

Sine series when function $f(x)$ is odd

$f(x) = \sum_{n=1}^{\infty} a_n sin(\frac{n\pi x}{l})$

$b_n = \frac{2}{l}\int_0^l f(x)sin(\frac{n\pi x}{l})dx$

[1]

[1]This documeny is created by http://physicscatalyst.com