Matrices

Question 1.
In the matrix

$$A=\begin{bmatrix} 2 & 5 & 19 & -7 \\ 35 & -2 & \frac{5}{2} & 12 \\ \sqrt{3} & 1 & -5 & 17 \end{bmatrix}$$

write:
(i) The order of the matrix,
(ii) The number of elements,
(iii) Write the elements a_{13}, a_{21}, a_{33}, a_{24}, a_{23}

Solution
(i) In the given matrix, the number of rows is 3 and the number of columns is 4. Therefore, the order of the matrix is 3 × 4.
(ii) Since the order of the matrix is 3 × 4, there are 3 × 4 = 12 elements in it.

(iii) a_{13}= 19, a_{21}= 35, a_{33}= −5, a_{24}= 12, a_{23} =5/2

Question 2.
If a matrix has 24 elements, what are the possible orders it can have? What, if it has 13 elements?

Solution
We know that if a matrix is of the order m × n, it has mn elements. Thus, to find all the possible orders of a matrix having 24 elements, we have to find all the ordered pairs of natural numbers whose product is 24.
The ordered pairs are: (1, 24), (24, 1), (2, 12), (12, 2), (3, 8), (8, 3), (4, 6), and (6, 4)

Hence, the possible orders of a matrix having 24 elements are:
1 × 24, 24 × 1, 2 × 12, 12 × 2, 3 × 8, 8 × 3, 4 × 6, and 6 × 4

If the matrix has 13 elements, then possible ordered pairs are (1, 13) and (13, 1) only.

Question 3.

If a matrix has 18 elements, what are the possible orders it can have? What, if it has 5 elements?

Solution

Like earlier question,
We must find all the ordered pairs of natural numbers whose product is 18.
The ordered pairs are: (1, 18), (18, 1), (2, 9), (9, 2), (3, 6,), and (6,3)
Hence, the possible orders of a matrix having 18 elements are:
1 × 18, 18 × 1, 2 × 9, 9 × 2, 3 × 6, and 6 × 3
(1, 5) and (5, 1) are the ordered pairs of natural numbers whose product is 5.
Hence, the possible orders of a matrix having 5 elements are 1 × 5 and 5 × 1.

Question 4.

Construct a 2 × 2 matrix, A = [aij], whose elements are given by:

i) $a_{ij} = (i+j)^2/2$
ii) $a_{ij} = i/j$
iii) $a_{ij} = (i+2j)^2/2$

Solution

The matrix will be given by the elements
a_{11}, a_{12}, a_{21}, a_{22}

$$\begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}$$

i) Substituting the values of i and j, we get the matrix as

$$\begin{bmatrix} 2 & \frac{9}{2} \\ \frac{9}{2} & 8 \end{bmatrix}$$

ii) Substituting the values of i and j, we get the matrix as

$$\begin{bmatrix} 1 & \frac{1}{2} \\ 2 & 1 \end{bmatrix}$$

iii) Substituting the values of i and j, we get the matrix as

$$\begin{bmatrix} \frac{9}{2} & \frac{25}{2} \\ 8 & 18 \end{bmatrix}$$

Question 5.
Construct a 3 × 4 matrix, whose elements are given by:
(i)a_{ij} = (1/2) |-3i+j|
(ii) *aij = 2i – j*

Solution

The matrix would be given by

$$\begin{bmatrix} a_{11} & a_{12} & a_{13} & a_{14} \\ a_{21} & a_{22} & a_{23} & a_{24} \\ a_{31} & a_{32} & a_{33} & a_{34} \end{bmatrix}$$

So i in 1,2,3
J in 1,2,3,4

i)
a_{11} = (1/2) |-3(1)+1|= 1
a_{12}= (1/2) |-3(1)+2|= ½
a_{13} = (1/2) |-3(1)+3|=0
a_{14} =(1/2) |-3(1)+4|=1/2

Similarly, can be calculated for other values

$$\begin{bmatrix} 1 & \frac{1}{2} & 0 & \frac{1}{2} \\ \frac{5}{2} & 2 & \frac{3}{2} & 1 \\ 4 & \frac{7}{2} & 3 & \frac{5}{2} \end{bmatrix}$$

ii)

$a_{11} = 2(1) - 1=1$

$a_{12} = 2(1) - 2=0$
$a_{13} = 2(1) - 3=-1$
$a_{14} = 2(1) - 4=-2$
Similarly, can be calculated for other values

$$\begin{bmatrix} 1 & 0 & -1 & -2 \\ 3 & 2 & 1 & 0 \\ 5 & 4 & 3 & 2 \end{bmatrix}$$

Question 6.
Find the values of *x*, *y* and *z* from the following equations:

(i) $$\begin{bmatrix} 4 & 3 \\ x & 5 \end{bmatrix} = \begin{bmatrix} y & z \\ 1 & 5 \end{bmatrix}$$

(ii) $$\begin{bmatrix} x+y & 2 \\ 5+z & xy \end{bmatrix} = \begin{bmatrix} 6 & 2 \\ 5 & 8 \end{bmatrix}$$

(iii) $$\begin{bmatrix} x+y+z \\ x+z \\ y+z \end{bmatrix} = \begin{bmatrix} 9 \\ 5 \\ 7 \end{bmatrix}$$

Solution

As the given matrices are equal, their corresponding elements are also equal.
So comparing the values, we can easily the find the values of x,y,z
i) Comparing the corresponding elements, we get:

y=4
z=3
x=1

ii) Comparing the corresponding elements, we get:

x+y=6
or x=6-y --(a)

5+z=5 or z=0
xy=8 --(b)

From (a) and (b)
y(6-y) =8
or
y^2 -6y+8 =0
or y =2 or 4

From (a)
x= 4 ,2

So the x,y,z values will be
(2,4,0) or (4,2,0)

iii)
Comparing the corresponding elements, we get:
x + y + z = 9 ... (a)
x + z = 5 ... (b)
y + z = 7 ... (c)
From (a) and (b), we have:
y + 5 = 9
or y = 4
Then, from (3), we have:
4 + z = 7
z = 3
Now
x + z = 5
x = 2
So values are
x = 2, y = 4, and z = 3

Question 7.
Find the value of *a*, *b*, *c* and *d* from the equation:

$$\begin{bmatrix} a-b & 2a+c \\ 2a-b & 3c+d \end{bmatrix} = \begin{bmatrix} -1 & 5 \\ 0 & 13 \end{bmatrix}$$

Solution

As the given matrices are equal, their corresponding elements are also equal.

So comparing the values, we can easily the find the values of a,b,c and d

a − b = −1 ... (1)
2a − b = 0 ... (2)
2a + c = 5 ... (3)
3c + d = 13 ... (4)

From (2), we have:
b = 2a
Then, from (1), we have:
a − 2a = −1
or a = 1
and b = 2
Now, from (3), we have:
2 ×1 + c = 5
Or c = 3
From (4) we have:
3 ×3 + d = 13
9 + d = 13
Or d = 4

a = 1, b = 2, c = 3, and d = 4

Question 8.
A = [*aij*]*m* × *n is* a square matrix, if
(A) *m* < *n* (B) *m* > *n* (C) *m* = *n* (D) None of these

Solution

The correct answer is C.

It is known that a given matrix is said to be a square matrix if the number of rows is equal to the number of columns. Therefore, is a square matrix, if m = n.

Question 9.
Which of the given values of *x* and *y* make the following pair of matrices equal

$$\begin{bmatrix} 3x+7 & 5 \\ y+1 & 2-3x \end{bmatrix}, \begin{bmatrix} 0 & y-2 \\ 8 & 4 \end{bmatrix}$$

a) x= -1/3, y=7
b) Not possible to find
c) y=7, x=-2/3
d) x=1/3 ,y=2/3

Solution Correct Answer is (B)

Explanation
For the matrices to be equal, each of the element should be equal to corresponding elements

3x+7 =0 or x= -7/3
5=y-2 or y = 7

2-3x=4 or x =-2/3

We find that on comparing the corresponding elements of the two matrices, we get two different values of x, which is not possible.
Hence, it is not possible to find the values of x and y for which the given matrices are equal

Question 10.
The number of all possible matrices of order 3 × 3 with each entry 0 or 1 is:
(A) 27

(B) 18

(C) 81

(D) 512

Solution

The correct answer is D.

The given matrix of the order 3 × 3 has 9 elements and each of these elements can be either 0 or 1.

Now, each of the 9 elements can be filled in two possible ways.

Therefore, by the multiplication principle, the required number of possible matrices is $2^9 = 512$