Probability Exercise 13.2

Question 1

If, P(A) =3/5 and P(B) =1/5 find P (A ∩ B) if A and B are independent events.

Solution

It is given that P(A) =3/5 and P(B) =1/5

A and B are independent events. Therefore,

P (A ∩ B) = P(A) P(B) = 3/25

Question 2

Two cards are drawn at random and without replacement from a pack of 52 playing cards. Find the probability that both the cards are black.

Solution

There are 26 black cards in a deck of 52 cards.

Let P (A) be the probability of getting a black card in the first draw

P(A) =26/52= 1/2

Let P (B) be the probability of getting a black card on the second draw.

Since the card is not replaced,

P(B) =25/51

Thus, probability of getting both the cards black =P(A) P(B) =25/102

Question 3:

A box of oranges is inspected by examining three randomly selected oranges drawn without replacement. If all the three oranges are good, the box is approved for sale, otherwise, it is rejected. Find the probability that a box containing 15 oranges out of which 12 are good and 3 are bad ones will be approved for sale.

Solution

Let A, B, and C be the respective events that the first, second, and third drawn orange is good.

Therefore, probability that first drawn orange is good, P (A) = 12/15

The oranges are not replaced.

Therefore, probability of getting second orange good, P (B) =11/14

Similarly, probability of getting third orange good, P(C) =10/13

The box is approved for sale, if all the three oranges are good. Thus, probability of getting all the oranges good

Therefore, the probability that the box is approved for sale is = 12/15 × 11/14 ×10/13= 44/91

Question 4

A fair coin and an unbiased die are tossed. Let A be the event 'head appears on the coin' and B be the event '3 on the die'. Check whether A and B are independent events or not.

Solution

If a fair coin and an unbiased die are tossed, then the sample space S is given by,

S= {(H, 1), (H, 2), (H, 3), (H, 4), (H, 5), (H, 6), (T, 1), (T, 2), (T, 3), (T, 4), (T, 5), (T, 6)}

Let A: Head appears on the coin

P(A) = 6/12 = 1/2

B: 3 on die

P(B) = 2/12= 1/6

Now

P (A ∩B) = 1/12

P(A) P(B)= 1/12

Therefore, A and B are independent events.

Question 5:

A die marked 1, 2, 3 in red and 4, 5, 6 in green is tossed. Let A be the event, 'the number is even,' and B be the event, 'the number is red'. Are A and B independent?

Solution

When a die is thrown, the sample space (S) is

S = {1, 2, 3, 4, 5, 6}

Let A: the number is even = {2, 4, 6}

B: the number is red = {1, 2, 3}

P(A) = 3/6 =1/2

P(B) = 3/6 =1/2

A ∩ B = {2}

P (A ∩ B) = 1/6

P(A) P(B) = ¼

As P (A ∩ B) ≠ P(A) P(B)

Therefore, A and B are not independent.

Question 6

Let E and F be events with P(E) =3/5 and P(F) = 3/10 and P (E ∩ F) =1/5. Are E and F

independent?

Solution

P(E) P(F) = 3/50

P (E ∩ F) =1/5

As P (E ∩ F) ≠ P(E) P(F)

Therefore, E and F are not independent.

Question 7

Given that the events A and B are such that and P(A) =1/2 and P (A ∪ B) = 3/5, P (B) = p. Find p

if they are (i) mutually exclusive (ii) independent.

Solution

It is given that

(i) When A and B are mutually exclusive, A ∩ B = Φ

P (A ∩ B) = 0

It is known that,

P(A∪B) =P(A)+P(B)–P(A∩B)
3/5 = ½ + p

p=1/10

(ii) When A and B are independent,

It is known that,

P(A∪B) =P(A)+P(B)–P(A∩B)
P(A∪B) =P(A)+P(B)–P(A∩B)

3/5= ½ + p – 1/2p

Or p =1/5

Question 8

Let A and B are independent events with P(A) =.3 and P(B) =.4, Find Following

(i) P (A ∩ B) (ii) P (A ∪ B)

(iii) P (A|B) (iv) P (B|A)

Solution

It is given that P (A) = 0.3 and P (B) = 0.4

(i) If A and B are independent events, then

P (A ∩ B) =P(A) P(B) = .12

(ii) It is known that,

P(A∪B) =P(A)+P(B)–P(A∩B)
So =.58

(iii) It is known that,

P (A|B) = P(A∩B) / P(B) = .12/.4 = .3

(iv) It is known that,

P (B|A) = P(A∩B) / P(A) = .12/.3 = .4

Question 9:

If A and B are two events such that, P(A) =1/4 and P(B) = ½ an P(A∩B) = 1/8 find P (not A and not B).

Solution

It is given that,

P (not on A and not on B) =P (A′ ∩ B′)

P (not on A and not on B) =P (A ∪B)′

= 1 – P (A∪ B)

=1 – (P(A)+P(B)–P(A∩B))
=3/8

Question 10:

Events A and B are such that P(A) = ½, P(B) =7/12 and P (not A or not B) =1/4. State whether A and B are independent?

Solution

It is given that

P(A) = ½, P(B) =7/12 and P (not A or not B) =1/4

Now

P (not A or not B) =1/4

P (A′ U B′) = ¼

P (A∩ B)′ = ¼ [As A′ U B′ and (A∩ B)′]

1 – P (A∩ B) =1/4

P (A∩ B) =3/4

Now for A and B to be independent

P (A ∩ B) = P(A) P(B)

3/4 ≠ 7/24

Therefore, A and B are independent events.

Question 11

Given two independent events A and B such that P (A) = 0.3, P (B) = 0.6. Find

(i) P (A and B) (ii) P (A and not B)

(iii) P (A or B) (iv) P (neither A nor B)

Solution

It is given that P (A) = 0.3 and P (B) = 0.6

Also, A and B are independent events.

(i) P (A and B) = P (A ∩ B) =P(A) P(B) = .18

(ii) P (A and not B) =P (A ∩ B')

=P(A) – P (A∩ B)

= .3 -.18 = .12

(iii) P (A or B) = P (A U B)

= P(A) + P(B) – P (A∩ B)

=.72

(iv) P (neither A nor B) = P (A' ∩ B')

= P [(A U B)']

= 1 – P (A U B)

=1 -.72 = .28

Question 12

A die is tossed thrice. Find the probability of getting an odd number at least once.

Solution

Probability of getting an odd number in a single throw of a die = 3/6 = 1/2

Similarly, probability of getting an even number =3/6= 1/2

Probability of getting an even number three times = (1/2) × (1/2) × (1/2) = 1/8

Therefore, probability of getting an odd number at least once

= 1 − Probability of getting an odd number in none of the throws

= 1 − Probability of getting an even number thrice

=1 -1/8 = 7/8

Question 13:

Two balls are drawn at random with replacement from a box containing 10 black and 8 red balls. Find the probability that

(i) both balls are red.

(ii) first ball is black and second is red.

(iii) one of them is black and other is red.

Solution

Total number of balls = 18

Number of red balls = 8

Number of black balls = 10

(i) Probability of getting a red ball in the first draw =8/18= 4/9

The ball is replaced after the first draw.

Probability of getting a red ball in the second draw =8/18 =4/9

Therefore, probability of getting both the balls red =4/9 × 4/9 = 16/81

(ii) Probability of getting first ball black =10/18 = 5/9

The ball is replaced after the first draw.

Probability of getting second ball as red =8/18 = 4/9

Therefore, probability of getting first ball as black and second ball as red =5/9 × 4/9 = 20/81

(iii) Probability of getting first ball as red =8/18= 4/9

The ball is replaced after the first draw.

Probability of getting second ball as black =10/18 = 5/9

Therefore, probability of getting first ball as black and second ball as red =4/9 × 5/9 = 20/81

Therefore, probability that one of them is black and other is red

= Probability of getting first ball black and second as red + Probability of getting first ball

red and second ball black

= 20/81 + 20/81 = 40/81

Question 14

Probability of solving specific problem independently by A and B are ½ and 1/3 respectively.

If both try to solve the problem independently, find the probability that

(i) the problem is solved (ii) exactly one of them solves the problem.

Solution

i. Probability that the problem is solved = P (A U B)

= P (A) + P (B) – P (A∩ B)

= P (A) + P (B) – P (A) P(B) [as they are independent]

=2/3

(ii) Probability that exactly one of them solves the problem is given by,

= P (A) P (B$^{'}$) + P (A$^{'}$) P (B)

= P(A) [1 – P(B)] + P(B) [1 – P(A)]

= ½ (1 – 1/3) + 1/3(1- ½)

=1/3 + 1/6 = 1/2

Question 15

One card is drawn at random from a well shuffled deck of 52 cards. In which of the

following cases are the events E and F independent?

(i) E: 'the card drawn is a spade'

F: 'the card drawn is an ace'

(ii) E: 'the card drawn is black'

F: 'the card drawn is a king'

(iii) E: 'the card drawn is a king or queen'

F: 'the card drawn is a queen or jack'

Solution

(i) In a deck of 52 cards, 13 cards are spades and 4 cards are aces.

P(E) = P (the card drawn is a spade) =13/52 =1/4

P(F) = P (the card drawn is an ace) =4/52 = 1/13

In the deck of cards, only 1 card is an ace of spades.

P (E ∩ F) = P (the card drawn is spade and an ace) = 1/52

P(E) × P(F) =1/52

P(E) × P(F) = P(E∩F)

Therefore, the events E and F are independent.

(ii) In a deck of 52 cards, 26 cards are black and 4 cards are kings.

P(E) = P (the card drawn is black) =26/52 = 1/2

P(F) = P (the card drawn is a king) =4/52 =1/13

In the pack of 52 cards, 2 cards are black as well as kings.

P (E ∩ F) = P (the card drawn is a black king) =2/52= 1/26

P(E) × P(F) =1/26

Therefore, the given events E and F are independent.

(iii) In a deck of 52 cards, 4 cards are kings, 4 cards are queens, and 4 cards are jacks.

P(E) = P (the card drawn is a king or a queen) =8/52 =2/13

P(F) = P (the card drawn is a queen or a jack) =8/52= 2/13

There are 4 cards which are king or queen and queen or jack.

P(E∩F) = P (the card drawn is a king or a queen, or queen or a jack)

= 4/52= 1/13

P(E) × P(F) =4/169

Therefore, the given events E and F are not independent.

Question 16

In a hostel, 60% of the students read Hindi newspaper, 40% read English newspaper

and 20% read both Hindi and English newspapers. A student is selected at random.

(a) Find the probability that she reads neither Hindi nor English newspapers.

(b) If she reads Hindi newspaper, find the probability that she reads English news paper

(c) If she reads English newspaper, find the probability that she reads Hindi newspaper.

Solution

Let H denote the students who read Hindi newspaper and E denote the students who

read English newspaper.

It is given that,

P(H) = .60

P(E) = .40

P (H∩ E) = .20

i. Probability that a student reads Hindi or English newspaper is,

P [(H U E)'] = 1 – P (H U E)

= 1- [P(H) + P(E) - P (H∩ E)]

=.20

(ii) Probability that a randomly chosen student reads English newspaper, if she reads

Hindi newspaper, is given by P (E|H).

P(E|H) = P (H∩ E)]/ P(H) = 1/3

(iii) Probability that a randomly chosen student reads Hindi newspaper, if she reads

English newspaper, is given by P (H|E).

P(H|E) = P (H∩ E)]/ P(E) = 1/2

Question 17:

The probability of obtaining an even prime number on each die, when a pair of dice is

rolled is

(A) 0

(B) 1/3

(C) 1/12

 (D) 1/36

Solution

When two dice are rolled, the number of outcomes is 36.

The only even prime number is 2.

Let Me be the event of getting an even prime number on each die.

E = {(2, 2)}

P(E) =1/36

Therefore, the correct answer is D.

Question 18:

Two events A and B will be independent, if

(A) A and B are mutually exclusive

(B) $P(A' B') = [1 - P(A)][1 - P(B)]$

(C) P(A) = P(B)

(D) P(A) + P(B) = 1

Solution

Two events A and B are said to be independent, if P(AB) = P(A) × P(B)

Consider the result given in alternative B.

P (A$'$ B$'$) = [1- P(A)] [1- P(B)]

P (A$'$ ∩ B$'$) = [1- P(A)] [1- P(B)]

P [(A U B)$'$] = [1- P(A)] [1- P(B)]

1 – P (A U B) = 1- P(A) - P(B) + P(A) P(B)

1 – P(A) – P(B) + P(A∩B) = 1- P(A) - P(B) + P(A) P(B)

Or P(A∩B) = P(A) × P(B)

This implies that A and B are independent

A. Let P (A) = m, P (B) = n, 0 < m, n < 1

A and B are mutually exclusive.

Then

A ∩B = {}

P (A ∩B) =0

But MN ≠0

So, it cannot be true

C. Let A: Event of getting an odd number on throw of a die = {1, 3, 5}

=1/2

B: Event of getting an even number on throw of a die = {2, 4, 6} =1/2

So, P(A) = P(B)

Here,

A ∩B = {}

P (A ∩B) =0

But 1/4 ≠0

So, it cannot be true

D. From the above example, it can be seen that,

P(A) + P(B) = ½ + ½ =1

However, it cannot be inferred that A and B are independent.

Thus, the correct answer is B.