

Relations and Functions

Important Things to Remember

Reflexive Relation

A relation in a set A is called reflexive relation if $(a,a) \in R$ for every element $a \in A$.

Symmetric Relation

A relation in a set A is called if $(a,b) \in R$ then $(b,a) \in R$ for all $a,b \in A$

Important Note: symmetry is a different kind of requirement than reflexivity. Reflexivity requires that certain pairs must be in R , namely all pairs of the form (a, a) for every element in A . However, symmetry only requires that if a pair (a, b) is in R , then the pair (b, a) must also be in R . But it is not required that pairs of the form (a, b) are in R unless the pair (b, a) is in R . Simply stated, you must have both pairs or neither

Transitive Relation

A relation R on a set A is called transitive if whenever (a, b) is in R and (b, c) is in R , then (a, c) is in R .

Important Note:

Note that transitivity, like symmetry, is possessed by a relation unless the stated condition is violated. So, unless you can find pairs (a, b) and (b, c) which are in R while (a, c) is not, then the relation is transitive. The empty relation is always transitive because it has no pairs to violate the condition

Equivalence Relation

A relation R on a Set A is called equivalence relation if R is reflexive, Symmetric and transitive

Question 1

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Determine whether each of the following relations are reflexive, symmetric and transitive:

(i) Relation R in the set $A = \{1, 2, 3, \dots, 13, 14\}$ defined as

$$R = \{(x, y) : 3x - y = 0\}$$

(ii) Relation R in the set N of natural numbers defined as

$$R = \{(x, y) : y = x + 5 \text{ and } x < 4\}$$

(iii) Relation R in the set $A = \{1, 2, 3, 4, 5, 6\}$ as

$$R = \{(x, y) : y \text{ is divisible by } x\}$$

(iv) Relation R in the set Z of all integers defined as

$$R = \{(x, y) : x - y \text{ is an integer}\}$$

(v) Relation R in the set A of human beings in a town at a time given by

(a) $R = \{(x, y) : x \text{ and } y \text{ work at the same place}\}$

(b) $R = \{(x, y) : x \text{ and } y \text{ live in the same locality}\}$

(c) $R = \{(x, y) : x \text{ is exactly 7 cm taller than } y\}$

(d) $R = \{(x, y) : x \text{ is wife of } y\}$

(e) $R = \{(x, y) : x \text{ is father of } y\}$

Solution

(i) $A = \{1, 2, 3, \dots, 13, 14\}$

$$R = \{(x, y) : 3x - y = 0\}$$

This can be written in another form as below

$$R = \{(1, 3), (2, 6), (3, 9), (4, 12)\}$$

R is not reflexive since $(1, 1), (2, 2), \dots, (14, 14) \notin R$.

Also, R is not symmetric as $(1, 3) \in R$, but $(3, 1) \notin R$.

Also, R is not transitive as $(1, 3), (3, 9) \in R$, but $(1, 9) \notin R$.

So, R is neither reflexive, nor symmetric, nor transitive.

(ii) $A = \{1, 2, 3, \dots, 13, 14\}$

$$R = \{(x, y): y = x + 5 \text{ and } x < 4\}$$

This can be written in another form as below

$$\{(1, 6), (2, 7), (3, 8)\}$$

It is seen that $(1, 1) \notin R$.

So, R is not reflexive.

$(1, 6) \in R$ But, $(1, 6) \notin R$.

So, R is not symmetric.

Now, since there is no pair in R such that (x, y) and $(y, z) \in R$, then

(x, z) cannot belong to R.

So, R is not transitive.

Hence, R is neither reflexive, nor symmetric, nor transitive.

(iii) $A = \{1, 2, 3, 4, 5, 6\}$

$$R = \{(x, y): y \text{ is divisible by } x\}$$

We know that any number (x) is divisible by itself.

$$(x, x) \in R$$

So, R is reflexive.

Now,

$$(2, 4) \in R \text{ [as 4 is divisible by 2]}$$

But,

$$(4, 2) \notin R. \text{ [as 2 is not divisible by 4]}$$

So, R is not symmetric.

Let $(x, y), (y, z) \in R$. Then, y is divisible by x and z is divisible by y.

Therefore, z is divisible by x.

So, R is transitive.

Hence, R is reflexive and transitive but not symmetric.

(iv) $R = \{(x, y) : x - y \text{ is an integer}\}$

Now, for every $x \in \mathbb{Z}$, $(x, x) \in R$ as $x - x = 0$ is an integer.

So, R is reflexive.

Now, for every $x, y \in \mathbb{Z}$ if $(x, y) \in R$, then $x - y$ is an integer.

Now $-(x - y)$ is also an integer.

So, $(y - x)$ is an integer.

Therefore $(y, x) \in R$

$\therefore R$ is symmetric.

Now,

Let (x, y) and $(y, z) \in R$, where $x, y, z \in \mathbb{Z}$.

Now $(x - y)$ and $(y - z)$ are integers.

Then $x - z = (x - y) + (y - z)$ is an integer.

So $(x, z) \in R$

Therefore, R is transitive.

Hence, R is reflexive, symmetric, and transitive.

(v) (a) $R = \{(x, y) : x \text{ and } y \text{ work at the same place}\}$

$(x, x) \in R$

$\therefore R$ is reflexive.

If $(x, y) \in R$, then x and y work at the same place.

$\Rightarrow y$ and x work at the same place.

$\Rightarrow (y, x) \in R$.

$\therefore R$ is symmetric.

Now, let $(x, y), (y, z) \in R$

$\Rightarrow x$ and y work at the same place and y and z work at the same place.

\Rightarrow x and z work at the same place.

$\Rightarrow (x, z) \in R$

$\therefore R$ is transitive.

Hence, R is reflexive, symmetric, and transitive.

(b) $R = \{(x, y) : x \text{ and } y \text{ live in the same locality}\}$

Clearly $(x, x) \in R$ as x and x is the same human being.

$\therefore R$ is reflexive.

If $(x, y) \in R$, then x and y live in the same locality.

\Rightarrow y and x live in the same locality.

$\Rightarrow (y, x) \in R$

$\therefore R$ is symmetric.

Now, let $(x, y) \in R$ and $(y, z) \in R$.

x and y live in the same locality and y and z live in the same locality.

x and z live in the same locality.

$(x, z) \in R$

$\therefore R$ is transitive.

Hence, R is reflexive, symmetric, and transitive.

(c) $R = \{(x, y) : x \text{ is exactly 7 cm taller than } y\}$

Now,

$(x, x) \notin R$

Since human being x cannot be taller than himself.

$\therefore R$ is not reflexive.

Now, let $(x, y) \in R$.

x is exactly 7 cm taller than y.

Then, y is not taller than x.

So, $(y, x) \notin R$

Therefore, R is not symmetric.

Now,

Let $(x, y), (y, z) \in R$.

Then x is exactly 7 cm taller than y and y is exactly 7 cm taller than z .

So, x is exactly 14 cm taller than z .

$(x, z) \notin R$

Therefore, R is not transitive.

Hence, R is neither reflexive, nor symmetric, nor transitive.

(d) $R = \{(x, y) : x \text{ is the wife of } y\}$

Now,

$(x, x) \notin R$

Since x cannot be the wife of herself.

$\therefore R$ is not reflexive.

Now, let $(x, y) \in R$

$\Rightarrow x$ is the wife of y .

Clearly y is not the wife of x .

$(y, x) \notin R$

Therefore, R is not transitive.

Let $(x, y), (y, z) \in R$

So x is the wife of y and y is the wife of z .

This case is not possible. Also, this does not imply that x is the wife of z .

$(x, z) \notin R$

Therefore, R is not transitive.

Hence, R is neither reflexive, nor symmetric, nor transitive.

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(e) $R = \{(x, y): x \text{ is the father of } y\}$

$(x, x) \notin R$

As x cannot be the father of himself.

Therefore, R is not reflexive.

Now, let $(x, y) \in R$.

So, x is the father of y .

Now y cannot be the father of x .

$(y, x) \notin R$

Therefore, R is not symmetric.

Now, let $(x, y) \in R$ and $(y, z) \in R$.

x is the father of y and y is the father of z .

Then clearly x is not the father of z . Infact x is grandfather of z

$(x, z) \notin R$

Therefore, R is not transitive.

Hence, R is neither reflexive, nor symmetric, nor transitive.

Question 2:

Show that the relation R in the set R of real numbers, defined as $R = \{(a, b): a \leq b^2\}$ is neither reflexive nor symmetric nor transitive.

Solution

$R = \{(a, b): a \leq b^2\}$

It can be observed that

$(1/3, 1/3)$ does not belong to this, as $1/3 > (1/3)^2$

Therefore, R is not reflexive.

Now, $(1, 4) \in R$ as $1 < 4^2$

But, 4 is not less than 1^2 .

So $(4, 1) \notin R$

Therefore, R is not symmetric.

Now,

$(3, 2), (2, 1.5) \in R$

(as $3 < 2^2 = 4$ and $2 < (1.5)^2 = 2.25$)

But, $3 > (1.5)^2 = 2.25$

$\therefore (3, 1.5) \notin R$

$\therefore R$ is not transitive.

Hence, R is neither reflexive, nor symmetric, nor transitive.

Question 3:

Check whether the relation R defined in the set $\{1, 2, 3, 4, 5, 6\}$ as $R = \{(a, b) : b = a + 1\}$ is reflexive, symmetric or transitive.

Solution

Let $A = \{1, 2, 3, 4, 5, 6\}$.

A relation R is defined on set A as:

$R = \{(a, b) : b = a + 1\}$

$\therefore R = \{(1, 2), (2, 3), (3, 4), (4, 5), (5, 6)\}$

We can find $(a, a) \notin R$, where $a \in A$.

For instance,

$(1, 1), (2, 2), (3, 3), (4, 4), (5, 5), (6, 6) \notin R$

$\therefore R$ is not reflexive.

It can be observed that $(1, 2) \in R$, but $(2, 1) \notin R$.

$\therefore R$ is not symmetric.

Now, $(1, 2), (2, 3) \in R$

But,

$(1, 3) \notin R$

$\therefore R$ is not transitive

Hence, R is neither reflexive, nor symmetric, nor transitive.

Question 4:

Show that the relation R in R defined as $R = \{(a, b): a \leq b\}$, is reflexive and transitive but not symmetric.

Answer:

$R = \{(a, b); a \leq b\}$

Clearly $(a, a) \in R$ as $a = a$.

$\therefore R$ is reflexive.

Now,

$(2, 4) \in R$ (as $2 < 4$)

But, $(4, 2) \notin R$ as 4 is greater than 2.

$\therefore R$ is not symmetric.

Now, let $(a, b), (b, c) \in R$.

Then,

$a \leq b$ and $b \leq c$

$\Rightarrow a \leq c$

$\Rightarrow (a, c) \in R$

$\therefore R$ is transitive.

Hence, R is reflexive and transitive but not symmetric.

Question 5:

Check whether the relation R in R defined as $R = \{(a, b): a \leq b^3\}$ is reflexive, symmetric or transitive.

Solution

$R = \{(a, b): a \leq b^3\}$

It is observed that

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$(1/3, 1/3)$ does not belong to this, as $1/3 > (1/3)^3$

Therefore, R is not reflexive.

Now,

$(1, 2) \in R$ (as $1 < 2^3 = 8$)

But,

$(2, 1) \notin R$ (as $2^3 > 1$)

\therefore R is not symmetric.

We have $(3, 3/2), (3/2, 6/5) \in R$

But

$(3, 6/5) \notin R$ [as $3 > (6/5)^3$]

Therefore, R is not transitive.

Hence, R is neither reflexive, nor symmetric, nor transitive.

Question 6:

Show that the relation R in the set $\{1, 2, 3\}$ given by $R = \{(1, 2), (2, 1)\}$ is symmetric but neither reflexive nor transitive.

Solution

Let $A = \{1, 2, 3\}$.

A relation R on A is defined as $R = \{(1, 2), (2, 1)\}$.

It is seen that $(1, 1), (2, 2), (3, 3) \notin R$.

Therefore, R is not reflexive.

Now, as $(1, 2) \in R$ and $(2, 1) \in R$, then R is symmetric.

Now, $(1, 2)$ and $(2, 1) \in R$

However,

$(1, 1) \notin R$

\therefore R is not transitive.

Hence, R is symmetric but neither reflexive nor transitive.

Question 7:

Show that the relation R in the set A of all the books in a library of a college, given by $R = \{(x, y) : x \text{ and } y \text{ have same number of pages}\}$ is an equivalence relation.

Solution

Set A is the set of all books in the library of a college.

$R = \{(x, y) : x \text{ and } y \text{ have the same number of pages}\}$

Now, R is reflexive since $(x, x) \in R$ as x and x has the same number of pages.

Let $(x, y) \in R \Rightarrow x$ and y have the same number of pages.

$\Rightarrow y$ and x have the same number of pages.

$\Rightarrow (y, x) \in R$

$\therefore R$ is symmetric.

Now, let $(x, y) \in R$ and $(y, z) \in R$.

$\Rightarrow x$ and y have the same number of pages and y and z have the same number of pages.

$\Rightarrow x$ and z have the same number of pages.

$\Rightarrow (x, z) \in R$

$\therefore R$ is transitive.

Hence, R is an equivalence relation.

Question 8:

Show that the relation R in the set $A = \{1, 2, 3, 4, 5\}$ given by $R = \{(a, b) : |a - b| \text{ is even}\}$, is an equivalence relation. Show that all the elements of $\{1, 3, 5\}$ are related to each other and all the elements of $\{2, 4\}$ are related to each other. But no element of $\{1, 3, 5\}$ is related to any element of $\{2, 4\}$.

Solution

$A = \{1, 2, 3, 4, 5\}$

$R = \{(a, b) : |a - b| \text{ is even}\}$,

For any element, $a \in A$, we have $|a - a| = 0$ (which is even).

$\therefore R$ is reflexive.

Let $(a, b) \in R$.

Now that means $|a - b|$ is even

Or $|-(b - a)|$ is even

Or $|b - a|$ is even

$\therefore R$ is symmetric.

Now, let $(a, b) \in R$ and $(b, c) \in R$.

Now that means $|a - b|$ and $|b - c|$ are even

Now $|a - c| = |(a - b) - (b - c)|$

So $(a, c) \in R$

Therefore, R is transitive.

Hence, R is an equivalence relation.

Now, all elements of the set $\{1, 3, 5\}$ are related to each other as all the elements of this subset are odd. Thus, the modulus of the difference between any two elements will be even.

Similarly, all elements of the set $\{2, 4\}$ are related to each other as all the elements of this subset are even. Also, no element of the subset $\{1, 3, 5\}$ can be related to any element of $\{2, 4\}$ as all elements of $\{1, 3, 5\}$ are odd and all elements of $\{2, 4\}$ are even. Thus, the modulus of the difference between the two elements (from each of these two subsets) will not be even.

Question 9:

Show that each of the relation R in the set $A = \{x \in \mathbf{Z} : 0 \leq x \leq 12\}$, given by

(i) $R = \{(a, b) : |a - b| \text{ is a multiple of } 4\}$

(ii) $R = \{(a, b) : a = b\}$

is an equivalence relation. Find the set of all elements related to 1 in each case.

Solution

$A = \{x \in \mathbf{Z} : 0 \leq x \leq 12\} = \{0, 1, 2, 3, 4, \dots, 12\}$

(i) $R = \{(a, b) : |a - b| \text{ is a multiple of } 4\}$

For any element, $a \in A$, we have $(a, a) \in R$ as $|a - a|$ is a multiple of 4.

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$\therefore R$ is reflexive.

Now, let $(a, b) \in R \Rightarrow |a-b|$ is a multiple of 4.

Now $|b-a| = |-(a-b)|$ which is multiple of 4

$\Rightarrow (b, a) \in R$

$\therefore R$ is symmetric.

Now, let $(a, b), (b, c) \in R$.

$|a-b|$ is multiple of 4

$|b-c|$ is multiple of 4

$|a-c| = |(a-b) - (b-c)|$

So $(a, c) \in R$

Therefore, R is transitive.

Hence, R is an equivalence relation.

The set of elements related to 1 is $\{1, 5, 9\}$ since

(ii) $R = \{(a, b) : a = b\}$

For any element, $a \in A$, we have $(a, a) \in R$, since $a = a$.

$\therefore R$ is reflexive.

Now, let $(a, b) \in R$.

$\Rightarrow a = b$

$\Rightarrow b = a$

$\Rightarrow (b, a) \in R$

$\therefore R$ is symmetric.

Now, let $(a, b) \in R$ and $(b, c) \in R$.

$\Rightarrow a = b$ and $b = c$

$\Rightarrow a = c$

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$\Rightarrow (a, c) \in R$

$\therefore R$ is transitive.

Hence, R is an equivalence relation.

The elements in R that are related to 1 will be those elements from set A which are equal to 1.

Hence, the set of elements related to 1 is $\{1\}$.

Question 10:

Given an example of a relation. Which is

- (i) Symmetric but neither reflexive nor transitive.
- (ii) Transitive but neither reflexive nor symmetric.
- (iii) Reflexive and symmetric but not transitive.
- (iv) Reflexive and transitive but not symmetric.
- (v) Symmetric and transitive but not reflexive.

Solution:

(i) Let $A = \{1, 2, 3\}$.

Define a relation R on A as $R = \{(1, 2), (2, 1)\}$.

Relation R is not reflexive as $(1, 1), (2, 2), (3, 3) \notin R$.

Now, as $(1, 2) \in R$ and $(2, 1) \in R$, R is symmetric.

$(1, 2), (2, 1) \in R$, but $(1, 1) \notin R$

Therefore, R is not transitive.

Hence, relation R is symmetric but not reflexive or transitive.

(ii) Consider a relation R in R defined as:

$R = \{(a, b) : a < b\}$

For any $a \in R$, we have $(a, a) \notin R$ since a cannot be strictly less than a itself. In fact, $a = a$.

Therefore, R is not reflexive.

Now,

$(2, 3) \in R$ (as $2 < 3$)

But, 3 is not less than 2.

$(3, 2) \notin R$

Therefore, R is not symmetric.

Now, let $(a, b), (b, c) \in R$.

$a < b$ and $b < c$

or $a < c$

$(a, c) \in R$

Therefore, R is transitive.

Hence, relation R is transitive but not reflexive and symmetric.

(iii) Let $A = \{1, 2, 3\}$.

Define a relation R on A as:

$A = \{(1, 1), (2, 2), (3, 3), (1, 2), (2, 1), (2, 3), (3, 2)\}$

Relation R is reflexive since for every $a \in A$, $(a, a) \in R$ i.e., $(1, 1), (2, 2), (3, 3) \in R$.

Relation R is symmetric since $(a, b) \in R \Rightarrow (b, a) \in R$ for all $a, b \in R$.

Relation R is not transitive as $(1, 2)$ and $(2, 3) \in R$ but $(1, 3) \notin R$

Hence, relation R is reflexive and symmetric but not transitive

iv)

(ii) Consider a relation R in R defined as:

$R = \{(a, b): a \geq b\}$

For any $a \in R$, we have $(a, a) \in R$, since $a = a$.

Therefore, R is reflexive.

Now,

$(3, 2) \in R$ (as $3 > 2$)

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But, 3 is not less than 2.

$(2, 3) \notin R$

Therefore, R is not symmetric.

Now, let $(a, b), (b, c) \in R$.

$a \geq b$ and $b \geq c$

or $a \geq c$

$(a, c) \in R$

Therefore, R is transitive.

Hence, relation R is transitive and reflexive but not symmetric.

(v) Let $A = \{1, 2\}$.

Define a relation R on A as:

$A = \{(1, 1), (1, 2), (2, 1)\}$

Relation R is not reflexive since for every $a \in A$, $(a, a) \notin R$

Relation R is symmetric since $(a, b) \in R \Rightarrow (b, a) \in R$ for all $a, b \in R$.

Relation R is transitive as $(1, 2)$ and $(2, 1) \in R$ and $(1, 1) \in R$

Hence, relation R is symmetric and transitive but not reflexive

Question 11

Show that the relation R in the set A of points in a plane given by $R = \{(P, Q): \text{distance of the point P from the origin is same as the distance of the point Q from the origin}\}$, is an equivalence relation.

Further, show that the set of all point related to a point $P \neq (0, 0)$ is the circle passing through P with origin as centre.

Solution

$R = \{(P, Q): \text{distance of point P from the origin is the same as the distance of point Q from the origin}\}$

Clearly, $(P, P) \in R$ since the distance of point P from the origin is always the same as the distance of the same point P from the origin.

Therefore, R is reflexive.

Now,

Let $(P, Q) \in R$.

That means the distance of point P from the origin is the same as the distance of point Q from the origin.

Clearly The distance of point Q from the origin is the same as the distance of point P from the origin.

$(Q, P) \in R$

Therefore, R is symmetric.

Now,

Let $(P, Q), (Q, S) \in R$.

The distance of points P and Q from the origin is the same and, the distance of points Q and S from the origin is the same.

So, the distance of points P and S from the origin is the same.

$(P, S) \in R$

Therefore, R is transitive.

Therefore, R is an equivalence relation.

The set of all points related to $P \neq (0, 0)$ will be those points whose distance from the origin is the same as the distance of point P from the origin.

In other words, if O $(0, 0)$ is the origin and $OP = k$, then the set of all points related to P is at distance k from the origin.

Hence, this set of points forms a circle with the centre as the origin and this circle passes through point P.

Question 12:

Show that the relation R defined in the set A of all triangles as $R = \{(T_1, T_2): T_1 \text{ is similar to } T_2\}$, is equivalence relation. Consider three right angle triangles T_1 with sides 3, 4, 5, T_2 with sides 5, 12, 13 and

T_3 with sides 6, 8, 10. Which triangles among T_1 , T_2 and T_3 are related?

Solution

$R = \{(T_1, T_2): T_1 \text{ is similar to } T_2\}$,

Now R is clearly reflexive since every triangle is similar to itself.

Now,

if $(T_1, T_2) \in R$, then

T_1 is similar to T_2 .

T_2 is similar to T_1 .

So, $(T_2, T_1) \in R$

Therefore, R is symmetric.

Now,

Let $(T_1, T_2), (T_2, T_3) \in R$.

T_1 is similar to T_2 and T_2 is similar to T_3 .

T_1 is similar to T_3 .

Therefore, R is transitive.

Thus, R is an equivalence relation.

Now, we can observe that:

The corresponding sides of triangles T_1 and T_3 are in the same ratio.

$$3/6 = 4/8 = 5/10 = 1/2$$

Then, triangle T_1 is similar to triangle T_3 .

So These two belongs to this relation

Question 13:

Show that the relation R defined in the set A of all polygons as $R = \{(P_1, P_2): P_1 \text{ and } P_2 \text{ have same number of sides}\}$, is an equivalence relation. What is the set of all elements in A related to the right angle triangle T with sides 3, 4 and 5?

Solution

$$R = \{(P_1, P_2) : P_1 \text{ and } P_2 \text{ have same the number of sides}\}$$

R is reflexive since $(P_1, P_1) \in R$ as the same polygon has the same number of sides with itself.

Let $(P_1, P_2) \in R$.

That means P_1 and P_2 have the same number of sides.

Or P_2 and P_1 have the same number of sides.

$$(P_2, P_1) \in R$$

Therefore, R is symmetric.

Now,

Let $(P_1, P_2), (P_2, P_3) \in R$.

P_1 and P_2 have the same number of sides.

P_2 and P_3 have the same number of sides.

So P_1 and P_3 have the same number of sides.

$$(P_1, P_3) \in R$$

Therefore, R is transitive.

Hence, R is an equivalence relation.

The elements in A related to the right-angled triangle (T) with sides 3, 4, and 5 are those polygons which have 3 sides (since T is a polygon with 3 sides).

Hence, the set of all elements in A related to triangle T is the set of all triangles.

Question 14:

Let L be the set of all lines in XY plane and R be the relation in L

defined as $R = \{(L_1, L_2) : L_1 \text{ is parallel to } L_2\}$. Show that R is an equivalence relation. Find the set of all lines related to the line $y = 2x + 4$.

Solution

$$R = \{(L_1, L_2) : L_1 \text{ is parallel to } L_2\}$$

Now R is reflexive as any line L_1 is parallel to itself i.e., $(L_1, L_1) \in R$.

Now,

Let $(L_1, L_2) \in R$.

Now that means L_1 is parallel to L_2 .

So L_2 is parallel to L_1 .

$(L_2, L_1) \in R$

Therefore, R is symmetric.

Now,

Let $(L_1, L_2), (L_1, L_3) \in R$.

$\Rightarrow L_1$ is parallel to L_2 . Also, L_2 is parallel to L_3 .

$\Rightarrow L_1$ is parallel to L_3 .

Therefore, R is transitive.

Hence, R is an equivalence relation.

The set of all lines related to the line $y = 2x + 4$ is the set of all lines that are parallel to the line $y = 2x + 4$.

Slope of line $y = 2x + 4$ is $m = 2$

It is known that parallel lines have the same slopes.

The line parallel to the given line is of the form $y = 2x + c$, where $c \in \mathbb{R}$.

Hence, the set of all lines related to the given line is given by $y = 2x + c$, where $c \in \mathbb{R}$.

Question 15:

Let R be the relation in the set $\{1, 2, 3, 4\}$ given by

$R = \{(1, 2), (2, 2), (1, 1), (4, 4), (1, 3), (3, 3), (3, 2)\}$.

Choose the correct answer.

(A) R is reflexive and symmetric but not transitive.

(B) R is reflexive and transitive but not symmetric.

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(C) R is symmetric and transitive but not reflexive.

(D) R is an equivalence relation.

Solution:

$$R = \{(1, 2), (2, 2), (1, 1), (4, 4), (1, 3), (3, 3), (3, 2)\}$$

It is seen that $(a, a) \in R$, for every $a \in \{1, 2, 3, 4\}$.

Therefore, R is reflexive.

It is seen that $(1, 2) \in R$, but $(2, 1) \notin R$.

Therefore, R is not symmetric.

Also, it is observed that $(a, b), (b, c) \in R \Rightarrow (a, c) \in R$ for all $a, b, c \in$

$\{1, 2, 3, 4\}$.

Therefore, R is transitive.

So, R is reflexive and transitive but not symmetric.

The correct answer is B.

Question 16:

Let R be the relation in the set N given by

$$R = \{(a, b) : a = b - 2, b > 6\}.$$

Choose the correct answer.

(A) $(2, 4) \in R$

(B) $(3, 8) \in R$

(C) $(6, 8) \in R$

(D) $(8, 7) \in R$

Solution

$$R = \{(a, b) : a = b - 2, b > 6\}$$

Now, since $b > 6$, $(2, 4) \notin R$

Also, as $3 \neq 8 - 2$, $(3, 8) \notin R$

And, as $8 \neq 7 - 2$

$(8, 7) \notin \mathbb{R}$

Now, consider $(6, 8)$.

We have $8 > 6$ and $6 = 8 - 2$.

Therefore, $(6, 8) \in \mathbb{R}$

The correct answer is C.