Scalar and vector fields

We know that many physical quantities like temperature, electric or gravitational field etc. have different values at different points in space for example electric field of a point charge is large near the charge and it decreases as we go farther away from the charge. So we can say that electric field here is the physical quantity that varies from point to in space and it can be expressed as a continuous function of position of point in that region of space .Now term field is defined as the region in which at every point some physical quantity (temperature, electric field etc.) has a value. So by field we mean both the region and the value of physical quantity in that region.

Scalar Fields

If we consider temperature within a solid then we have a scalar field since temperature is a scalar quantity and by scalar field we mean that there are a set of values of a scalar that must be assigned throughout a continuous region of space. Again this field may be time dependent if heat is being supplied to the solid. Physical quantities like electric potential, gravitational potential, temperature, density etc. are expressed in the form of scalar fields.

Graphically scalar fields can be represented by contours which are imaginary surfaces drawn through all points for which field has same value. For temperature field the contours are called isothermal surfaces or isotherms. In case of electrostatics, if we put a point charge at any place, then electric potential around it will depend on the position of the point. Since electric potential is a scalar quantity, the field around the charge will be known as scalar potential field. If all such points at which potential is constant is joined by a surface then such a surface is called equipotential surface. Such surfaces are also called level surfaces, each level surface having its own constant value. Two level surfaces cannot cut each other because if they do so then scalar values corresponding to both must hold along their common line which contradicts our definition. Thus scalar point functions i.e. $\phi(x, y, z)$ are single valued functions.

Vector Fields

Like scalar fields we also have vector fields in which a vector is given for each point in space. As an example consider a fluid flowing along a tube of varying cross-section. In this case if we specify the fluid velocity at each point, we obtain a vector field, which may be dependent on time if pressure difference across the tube is varied with time.

The intensity of electric field, magnetic field and gravitational field etc. are the examples of a vector field. A vector field is represented at every point by a continuous vector function say $\vec{A}(x, y, z)$.At any specific point of the field, the function $\vec{A}(x, y, z)$ gives a vector of definite magnitude and direction, both of which changes continuously from point to point throughout the field region.

Graphically vector fields are represented by lines known as field or flux lines. These lines are drawn in the field in such a way that tangent at any point of the line gives the direction of vector field at that point. To express the magnitude of vector field at any point first draw an infinitesimal area perpendicular to the field line. The number of field lines passing through this area element gives the magnitude of vector field. One more important thing here to note is that, the lines representing vector fields cannot cross, because if they cross they would give non unique field direction at the point of interaction.

[1]

[1]This document is created by http://physicscatalyst.com