Trigonometric Functions Formative assessment Mathematics

**Fill in the blank**

(a) The value of the trigonometric function $\csc(-1410^\circ)$ is ............ (2,3)

(b) The value of the $\sin^2 \frac{\pi}{6} + \cos^2 \frac{\pi}{3} - \tan^2 \frac{\pi}{4}$ is ...................(0.5/-0.5)

(c) The value of $\sin 75^\circ$ is ......

(d) The radian measure of the angle $25^\circ$ is ............

Solution

a) 2

b) -0.5

**True or False statement**

(1) We can find an value of $\theta$ for which $\cos \theta = 2$

(2) $\pi$ radian is equal to 180 degrees

(3) $\tan \theta + \cot \theta = \frac{1}{\sin \theta \cos \theta}$

(4) The range of the secant function is defined as $(\infty, -1) \cup [1, \infty)$

(5) $25^\circ = \frac{\pi}{36}$
(6) \( \sin \left( -\frac{11\pi}{3} \right) = \frac{1}{\sqrt{2}} \)
(7) \( \cos 3x + \cos x - \cos 2x = 0 \) for \( x = \frac{\pi}{3} \)
(8) \( \sin^4(x) - \cos^4(x) = 2\sin^2(x) - 1 \)

**Solutions**
1) F
2) T
3) T
4) T
5) T
6) F
7) T
8) T

**Subjective Questions**

Find the domain and range of the following trigonometric function:
(1) \( y = 3\sin(x) \)
(2) \( y = \frac{1}{2}\cos(x) \)
(3) \( y = 2\tan(x) \)
(4) \( y = \cos(x) + 1 \)
(5) \( y = \sqrt{3 - \sin(x)} \)

**Linked Type comprehension**

If \( \cos \theta = \frac{1}{3} \) and \( \theta \) is fourth quadrant. Find the following
(i) Write all the trigonometric identities
(ii) Find out the sign of the other basic quantities like \( \sin, \tan, \sec, \cosec \) and \( \cot \)
(iii) The value of \( \sin \theta + \cos \theta \)
(iv) The value of \( \tan \theta + \cot \theta \)
(v) Find the value of \( \sec \theta \)

**Subjective question**

a) Prove that
\[
(\cos x + \cos y)^2 + (\sin x - \sin y)^2 = 4\cos^2 \frac{x+y}{2}
\]

b) Prove that
\[
\frac{\sin(A+B)+\sin(A-B)}{\sin(A+B)-\sin(A-B)} = \tan AcotB
\]
c) Prove that
\((\sin 3x + \sin x) \sin x + (\cos 3x - \cos x) \cos x = 0\)

Solution (a)

\[
LHS = \left(\frac{x+y}{2}\right)^2 + \left(\frac{x-y}{2}\right)^2 = 4 \cos^2 \frac{x+y}{2}
\]

\[
= \cos^2 x + \cos^2 y + 2 \cos x \cos y + \sin^2 x + \sin^2 y - 2 \sin x \sin y
\]

\[
= (\sin^2 x + \sin^2 x) + (\sin^2 y + \sin^2 y) + 2(\cos x \cos y - \sin x \sin y)
\]

\[
= 1 + 1 + 2 \cos (x+y) + \left[\cos (A + B) = (\cos A \cos B - \sin A \sin B)\right]
\]

\[
= 2 + 2 \cos (x+y)
\]

\[
= 2\left[1 + \cos (x+y)\right]
\]

\[
= 2\left[1 + 2 \cos^2 \left(\frac{x+y}{2}\right) - 1\right]
\]

\[
= 4 \cos^2 \left(\frac{x+y}{2}\right) = R.H.S.
\]

b) We know that
\[
\sin (A+B) = \sin (A) \cos (B) + \sin (B) \cos (A)
\]

\[
\sin (A-B) = \sin (A) \cos (B) - \sin (B) \cos (A)
\]

So substituting these values we get \(LHS = RHS\)

\[
L.H.S.
\]

\[
= (\sin 3x + \sin x) \sin x + (\cos 3x - \cos x) \cos x
\]

\[
= \sin (3x) \sin (x) + \sin^2 (x) + \cos (3x) \cos (x) - \cos^2 (x)
\]

\[
= \sin (3x) \sin (x) + \cos (3x) \cos (x) + \sin^2 (x) - \cos^2 (x)
\]

\[
= \sin (3x) \sin (x) + \cos (3x) \cos (x) - (\cos^2 (x) - \sin^2 (x))
\]

\[
= \cos (3x-x) - \cos (2x)
\]

\[
= 0
\]

**Subjective Questions**

If \(\theta\) lies between 180° and 270° and \(\sin \theta = -\frac{\sqrt{3}}{3}\),

Find following

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a) What is value of Sin(\(\theta/2\))
b) What is value of cos(\(\theta/2\))
c) What is value of tan(\(\theta/2\))

Solution:

The value of \(\theta/2\) will be expressed in for cos \(\theta\), Now since \(\theta\) lies in third quadrant, it is negative. Also

\[
\cos \theta = -\sqrt{1 - \sin^2 \theta} = -\frac{2}{3}
\]

\(180 < \theta < 270\)

\(90 < \theta/2 < 135\)

Hence \(\theta/2\) is a second quadrant angle so Sin(\(\theta/2\)) is positive and cos(\(\theta/2\)) is negative.

And tan(\(\theta/2\)) is negative.

Now half angles formula’s are as

\[
\sin \left(\frac{\theta}{2}\right) = \pm \sqrt{\frac{1-\cos \theta}{2}}
\]

We will use the positive sign

\[
\sin \left(\frac{\theta}{2}\right) = \sqrt{\frac{1-\cos \theta}{2}}
\]

Now half angles formula’s are as

\[
\cos \left(\frac{\theta}{2}\right) = \pm \sqrt{\frac{1+\cos \theta}{2}}
\]

We will use the negative sign

\[
\cos \left(\frac{\theta}{2}\right) = -\sqrt{\frac{1+\cos \theta}{2}}
\]

Now half angles formula’s are as

\[
\tan \left(\frac{\theta}{2}\right) = \pm \sqrt{\frac{1-\cos \theta}{1+\cos \theta}}
\]

We will use the negative sign

\[
\tan \left(\frac{\theta}{2}\right) = -\sqrt{\frac{1-\cos \theta}{1+\cos \theta}}
\]

So substituting the values, we get

\[
\sin \frac{\theta}{2} = \frac{\sqrt{30}}{6}
\]

\[
\cos \frac{\theta}{2} = -\frac{\sqrt{6}}{6}
\]

\[
\tan \frac{\theta}{2} = -\sqrt{5}
\]