Trigonometric Functions Formative assessment Mathematics

Fill in the blank

(a) The value of the trigonometric function cosec (-1410°) is ..... (2,3)
(b) The value of the $sin^2\frac{\pi}{6} + cos^2\frac{\pi}{3} - tan^2\frac{\pi}{4}$ is .....(.5/-.5)

(c) The value of sun75° is .....

(d) The radian measure of the angle 25° is .....

Solution
a)2
b) -.5

True or False statement

(1) We can find an value of θ for which cos θ =2
(2) π radian is equal to 180 degrees
(3) $\tan\theta + \cot\theta = \frac{1}{sin\theta cos\theta}$

(4) The range of the secant function is defined as (∞,-1) ∪ [1, ∞)

(5) 25^0=5 π/36

(6) $\sin\left(-\frac{11\pi}{3}\right) = \frac{1}{\sqrt{2}}$
(7) cos3x+cosx −cos2x=0 for $x = \frac{\pi}{3}$
(8) $\sin^4(x)-\cos^4(x)=2\sin^2(x)-1$

Solutions

1) F
2) T
3) T
4) T
5) T
6) F
7) T
8) T

Subjective Questions

Find the domain and range of the following trigonometric function:
(1) y=3sin(x)
(2) y=1/2cos(x)
(3) y=2tanx
(4) y=cos(x) +1
(5) $y = \sqrt{3 - \sin(x)}$

Linked Type comprehension

If Cos θ=1/3 and θ is fourth quadrant. Find the following
(i) Write all the trigonometric identities
(ii) Find out the sign of the other basic quantities like sin,tan,sec,cosec and cot
(iii) The value of sin θ+ cos θ
(iv) The value of tan θ + cot θ
(v) Find the value of sec θ

Subjective question

a) Prove that

$$(\cos x + \cos y)^2 + (\sin x - \sin y)^2 = 4\cos^2\frac{x+y}{2}$$

b) Prove that
$\frac{\sin(A+B)+\sin(A-B)}{\sin(A+B)-\sin(A-B)} = tanAcotB$

c) Prove that

(sin 3x + sin x) sin x + (cos 3x – cos x) cos x = 0

Solution (a)

LHS = $(\cos x+\cos y)^2+(\sin x-\sin y)^2=4\cos^2\frac{x+y}{2}$

$=\cos^2 x+\cos^2 y+2\cos x\cos y+\sin^2 x+\sin^2 y-2\sin x\sin y$

$=(\cos^2 x+\sin^2 x)+(\cos^2 y+\sin^2 y)+2(\cos x\cos y-\sin x\sin y)$

$=1+1+2\cos(x+y)$ $\quad[\cos(A+B)=(\cos A\cos B-\sin A\sin B)]$

$=2+2\cos(x+y)$

$=2[1+\cos(x+y)]$

$=2\left[1+2\cos^2\left(\frac{x+y}{2}\right)-1\right]$ $\quad[\cos 2A=2\cos^2 A-1]$

$=4\cos^2\left(\frac{x+y}{2}\right)=\text{R.H.S.}$

b) We know that

$\sin(A+B)=\sin(A)\cos(B)+\sin(B)\cos(A)$

$\sin(A-B)=\sin(A)\cos(B)-\sin(B)\cos(A)$

So substituting these values we get LHS=RHS

c)

L.H.S.

= (sin 3x + sin x) sin x + (cos 3x – cos x) cos x

=sin(3x) sin(x) + $\sin^2(x)$ + cos(3x)cos(x) –$\cos^2(x)$

= sin(3x) sin(x)+ cos(3x)cos(x)+ $\sin^2(x)$ –$\cos^2(x)$

= sin(3x) sin(x)+ cos(3x)cos(x)-($\cos^2(x)$- $\sin^2(x)$)

=cos(3x-x)-cos(2x)

=0

Subjective Questions

If θ lies between 180˚ and 270˚ and $\sin\theta=-\frac{\sqrt{5}}{3}$,

Find following

a) What is value of Sin(θ/2)
b) What is value of cos(θ/2)
c) What is value of tan(θ/2)

Solution:

The value of θ/2 will be expressed in for cos θ, Now since θ lies in third quadrant,it is negative Also

$$cos\theta = -\sqrt{1 - sin^2\theta} = -2/3$$

180 < θ < 270
90 < θ/2 < 135
Hence θ/2 is a second quadrant angle so Sin(θ/2) is positive and cos(θ/2) And tan(θ/2) is negative

Now half angles formula's are as

$$\sin\left(\frac{\theta}{2}\right) = \mp\sqrt{\frac{1-cos\theta}{2}}$$

We will use the positive sign

$$\sin\left(\frac{\theta}{2}\right) = \sqrt{\frac{1-cos\theta}{2}}$$

Now half angles formula's are as

$$\cos\left(\frac{\theta}{2}\right) = \mp\sqrt{\frac{1+cos\theta}{2}}$$

We will use the negative sign

$$\cos\left(\frac{\theta}{2}\right) = -\sqrt{\frac{1+cos\theta}{2}}$$

Now half angles formula's are as

$$\tan\left(\frac{\theta}{2}\right) = \mp\sqrt{\frac{1-cos\theta}{1+cos\theta}}$$

We will use the negative sign

$$\tan\left(\frac{\theta}{2}\right) = -\sqrt{\frac{1-cos\theta}{1+cos\theta}}$$

So substituting the values, we get

$$sin\frac{\theta}{2} = \frac{\sqrt{30}}{6}$$

$$cos\frac{\theta}{2} = -\frac{\sqrt{6}}{6}$$

$$tan\frac{\theta}{2} = -\sqrt{5}$$