Vector Algebra in short

A vector is a quantity that requires both a magnitude (= 0) and a direction in space it can be represented by an arrow in space for example electrostatic force, electrostatic field etc. In symbolic form we will represent vectors by bold letters. In component form vector A is written as

A = A_x**i** + A_y**j** + A_z**k**

ADDITION OF VECTORS

Two vectors A and B can be added together to give another resultant vector C.

C = **A** + **B**

SUBTRACTION OF VECTORS

Two vectors A and B can be subtracted to give another resultant vector D.

D = **A** - **B** = **A** + (-**B**)

SCALAR MULTIPLICATION OF VECTOR

When we multiply any vector A with any scalar quantity 'n' then it's direction remains unchanged and magnitude gets multiplied by 'n'. Thus,

n(**A**) = n**A**

Scalar multiplication of vectors is distributive i.e.,

n(**A** + **B**) = n**A** + n**B**

DOT PRODUCT OF VECTORS

Dot product of two vectors A and B is defined as the product of the magnitudes of vectors A and B and the cosine of the angle between them when both te vectors are placed tail to tail.

Dot product is represented as **A.B** thus,

A.B = |A| |B| $\cos\theta$

where θ is the angle between two vectors.

Result of dot product of two vectors is a scalar quantity.

Dot product is commutative : **A.B** = **B.A**

Dot product is distributive : **A** . (**B+C**) = **A.B** + **A.C** also **A.A** = $|A|^2$

CROSS PRODUCT OF TWO VECTORS

Cross product or vector product of two vectors A and B is defined as

A x **B** = |A| |B| $\sin\theta$ n̂

where n̂ is the unit vector pointing in the direction perpendicular to the plane of both A and B.

Result of vector product is also a vector quantity.

Cross product is distributive i.e., **A** x (**B** + **C**) = (**A** x **B**) + (**A** x **C**) but not commutative and the cross product of two parallel vectors is zero.

VECTOR ADDITION
In component form addition of two vectors is

C = $(A_x + B_x)$**i** + $(A_y + B_y)$**j** + $(A_y + B_y)$**k**

where,

A = (A_x, A_y, A_z) and B = (B_x, B_y, B_z)

Thus in component form resultant vector **C** becomes,

$C_x = A_x + B_x$

$C_y = A_y + B_y$

$C_z = A_z + B_z$

SUBTRACTION OF TWO VECTORS
In component form subtraction of two vectors is

D = $(A_x - B_x)$**i** + $(A_y - B_y)$**j** + $(A_y - B_y)$**k**

where,

A = (A_x, A_y, A_z) and B = (B_x, B_y, B_z)

Thus in component form resultant vector **D** becomes,

$D_x = A_x - B_x$

$D_y = A_y - B_y$

$D_z = A_z - B_z$

NOTE:- Two vectors add or subtract like components.

DOT PRODUCT OF TWO VECTORS
A.B = $(A_x i + A_y j + A_z k) . (B_x i + B_y j + B_z k)$

= $A_x B_x + A_y B_y + A_z B_z$.

Thus for calculating the dot product of two vectors, first multiply like components, and then add.

CROSS PRODUCT OF TWO VECTORS
A x B = $(A_x$**i** + A_y**j** + $A_z k) x (B_x i + B_y j + B_z$**k**)

= $(A_y B_z - A_z B_y)$**i** + $(A_z B_x - A_x B_z)$**j** + $(A_x B_y - A_y B_x)$**k**.

Cross product of two vectors is itself a vector.

To calculate the cross product, form the determinant whose first row is x, y, z, whose second row is A (in component form), and whose third row is B.

VECTOR TRIPPLE PRODUCT

Vector product of two vectors can be made to undergo dot or cross product with any third vector.

(a) Scalar triple product:-

For three vectors A, B, and C, their scalar triple product is defined as

A . (B x C) = B . (C x A) = C . (A x B)

obtained in cyclic permutation. If **A** = (A_x, A_y, A_z) , **B** = (B_x, B_y, B_z) , and **C** = (C_x, C_y, C_z) then **A . (B x C)** is the volume of a parallelepiped having A, B, and C as edges and can easily obtained by finding the determinant of the 3 x 3 matrix formed by **A**, **B**, and **C**.

(b) Vector Triple Product:-

For vectors A, B, and C, we define the vector tipple product as

A x (B x C) = B(A . C) – C(A – B)

Note that

(A . B)C not equal to **A(B . C)**

but

(A . B)C = C(A . B).